DAVIES & PECK'S

UNITED COURSE

ELEMENTARY ARITHMETIC

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ORAL AND WRITTEN

BY

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DAVIES AND PECK'S

SHORT COURSE IN MATHEMATICS

IN FOUR BOOKS.

ELEMENTARY ARITHMETIC.

COMPLETE ARITHMETIC.

MANUAL OF ALGEBRA.

MANUAL OF GEOMETRY.

PREFACE.

THIS work is designed as the Introductory Volume of the Two Book Course of Davies and Peck. It is especially adapted to beginners. It is believed that the subjects are treated in such a manner as to interest and awaken the attention of the young.

In preparing the work, three objects have been constantly kept in view.

1. To make it educational.

2. To make it practical.

3. To adapt it to the capacity of any child whose mind is sufficiently mature to commence the study of arithmetic.

To attain these objects, every new subject has been introduced by an inductive process, and the idea thus developed has been expressed in the form of a definition. The methods and rules have been deduced from practical operations and enforced by familiar illustrations. To direct the attention to important principles, leading test questions have been freely introduced.

In determining the subjects to be included, and the space to be assigned to each, the author has been guided by a consideration of the natural development of the

mental faculties. The book may be said to consist of five parts. The first part contains simple, familiar Lessons in Numbers. The second part contains the Fundamental Operations followed by General Principles and Properties of Numbers. The third contains Fractions, in which great pains have been taken to render the work intelligible to young students. Currency and the Metric System follow, because of their intimate relation to Decimal Fractions. The fourth contains Compound Numbers and Reduction. The fifth, Percentage and its applications.

The logical development of principles, the systematic arrangement of the subjects, the copiousness and variety of exercises will, it is believed, greatly aid the teacher in exciting the interest of the pupil.

Teachers who desire to give a more extended drill in the simplest operations, are referred to "PECK'S FIRST LESSONS IN NUMBERS."

To facilitate references, a complete Index to the Subjects and Definitions is inserted at the end of the volume.

The author takes great pleasure in acknowledging his obligations to many teachers who have favored him with suggestions and criticisms. But more than a passing acknowledgment is due to Prof. JOHN DUNLAP, whose long experience and superior ability as a Teacher have enabled him to render much valuable assistance in the preparation of this work.

CONTENTS.

FORMATION OF NUMBERS.

LESSON I.

COUNTING.

Look at the picture and count the objects named below.

How many houses?
How many horses?
How many sail-boats?
How many high trees?
How many boats?
How many boys at play?
How many windows in front?
How many small trees?
How many birds?
How many children at play?

LESSON II.

WRITING NUMBERS. 1 TO 10.*

Write the word that tells how many houses there are in the picture. *One.* One is a **Unit.**

Write the *word* that tells how many horses. *Two.* How many ones, or units, in two?

Write the *word* that tells how many persons there are in the carriage. *Three.* How many units in three?

How many units, or ones, in four? In five? In six? In seven? In eight? In nine? How many in ten?

One, two, three, four, five, etc., are called *numbers.*

A **Number** is one or more things of the same kind.

What number tells how many girls there are on the grounds? What is the number of boys?

Thus far we have used *words* to express numbers; we may also use **Figures.**

The number of houses may be written *one* or *1;* the number of horses, *two* or *2;* the number of sail-boats, *three* or *3;* the number of girls, *four* or *4;* number of boats on the lake, *five* or *5;* number of boys, *six* or *6;* number of windows, *seven* or *7;* number of small trees, *eight* or *8;* number of birds, *nine* or *9.*

We use one more figure, *0.* It is called *naught,* and standing alone expresses no number, but is used with other figures to express numbers.

These are all the figures in use. How many are there? Write the ten figures; thus,

1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

Read the following figures: 3, 2, 1; 4, 5, 6; 9, 8, 7, 0.

Which is the least number? The greatest?

* See Picture, page 7.

LESSON III.

NUMBERS FROM 10 TO 20.

How many figures do we use to express numbers?

The number *ten* is written by means of figures, thus, *10.*

This is one ten. How many units in one ten? Write ten.

How many boys are snow-balling?

We write the number by means of figures, thus, *11.*

The right-hand figure is 1 unit. The second figure from the right is 1 ten. Eleven is one ten and one unit. How many units in ten? How many units in eleven?

We will write, by means of figures, all the numbers from 10 to 20:

10,	*11,*	*12,*	*13,*	*14,*	*15,*
ten,	*eleven,*	*twelve,*	*thirteen,*	*fourteen,*	*fifteen,*
16,	*17,*	*18,*	*19,*	*20.*	
sixteen,	*seventeen,*	*eighteen,*	*nineteen,*	*twenty.*	

In these numbers, how many figures are used?

LESSON IV.

NUMBERS FROM 20 TO 30.

Two tens and 1 unit are twenty-one,	21.
Two tens and 2 units are twenty-two,	22.
Two tens and 3 units are twenty-three,	23.
Two tens and 4 units are twenty-four,	24.
Two tens and 5 units are twenty-five,	25.
Two tens and 6 units are twenty-six,	26.
Two tens and 7 units are twenty-seven,	27.
Two tens and 8 units are twenty-eight,	28.
Two tens and 9 units are twenty-nine,	29.

How many are one ten and 1? One ten and 2? One ten and 3? One ten and 4? One ten and 5? One ten and 6? One ten and 7? One ten and 8? One ten and 9? Two tens are how many? Two tens and 1? Two tens and 2? 2 tens and 3? 2 tens and 4? 2 tens and 5? 2 tens and 6? 2 tens and 7? 2 tens and 8? 2 tens and 9?

Write, in figures, eight, six, seventeen, nineteen, twenty-one, twenty-five, twenty-six, twenty-nine.

Write as one number, 1 ten and 7, 2 tens and 3, 1 ten and 8, 2 tens and 5, 2 tens and 7, 1 ten and 4.

Read the following numbers, 22, 19, 18, 29, 27, 16, 23, 21, 11, 12, 15, 24, 28.

One ten and one unit are how many? Two tens and two units are how many? Two tens and six units?

How many are one ten and six units? Two tens and six units? Write two tens and five units as one number.

Write three tens and six units; two tens and seven units; two tens and eight units; two tens and nine units.

LESSON V.

NUMBERS FROM 30 TO 100.

Three tens are thirty, 30. Four tens are forty, 40. Five tens are fifty, 50. Six tens are sixty, 60. Seven tens are seventy, 70. Eight tens are eighty, 80. Nine tens are ninety, 90. Ten tens are one-hundred, 100.

In writing tens we use two figures, and the second figure from the right tells how many tens we have written.

In writing 100 we use three figures, and the third figure from the right shows how many hundreds we have written.

If we use 2 instead of 1 in the third place, we have 200, (two hundred). If we use 3, we have 300; if 4, 400, and so on.

Write the numbers between 30 and 40:

Thus, *31, 32, 33, 34, 35, 36, 37, 38, 39.*

Write the numbers between 40 and 50; between 50 and 60; between 60 and 70; between 70 and 80; between 80 and 90; between 90 and 100.

Read the following numbers, 11, 14, 29, 23, 28, 31, 40, 37, 36, 42, 45, 49, 51, 53, 57, 62, 65, 69, 70, 75, 78, 82, 90, 87, 93, 71, 98, 86, 99, 100.

Four tens and 1 are how many? 4 tens and 3?

Five tens and 6 are how many? 5 tens and 7?

Six tens and 9? 6 tens and 5? 6 tens and 8?

Seven tens and 4 are how many? 7 tens and 8? 7 tens and 9?

Eight tens and 5 are how many? 8 tens and 6? 8 tens and 7?

Nine tens are how many? 9 tens and 1? 9 tens and 2? 9 tens and 3? 9 tens and 9? 9 tens and 7?

LESSON VI.

INCREASING AND DIMINISHING BY 1.

1. How many eggs are two eggs and one egg? 2 and 1, are how many? 1 and 2, are how many?

2. If we take one egg away from three eggs, how many eggs will be left? 2 from 3 leaves how many?

3. Three sheep and one sheep are how many sheep? 3 and 1, are how many? 1 and 3, are how many?

4. If we take 1 sheep away from 4 sheep, how many sheep will be left? 3 from 4 leaves how many?

5. Four birds and one bird are how many birds? 4 and 1 are how many? 1 and 4 are how many? One sheep and four sheep are how many?

6. If we take one bird from five birds, how many birds will be left? 4 from 5 leaves how many?

7. How many boys are five boys and one boy? How many are 5 and 1? 1 and 5 are how many?

8. If we take one apple from six apples, how many apples will be left? 1 from 6 leaves how many? 5 from 6 leaves how many?

9. Six chairs and one chair are how many chairs?

10. If we take one book from seven books, how many books are left? 1 from 7 leaves how many?

LESSON VII.

INCREASING AND DIMINISHING BY 2.

1. Two apples and two apples are how many apples?

2. If we take 2 apples from 4 apples, how many apples will be left?

3. Three sheep and two sheep are how many sheep? 3 and 2 are how many? 2 and 3 are how many?

4. If we take 2 sheep from 5 sheep, how many sheep will be left? 2 from 5 leaves how many? 3 from 5 leaves how many?

5. How many cherries are 4 cherries and 2 cherries? 4 and 2 are how many? 2 and 4 are how many?

6. If we take 2 cherries from 6 cherries, how many cherries will be left? 2 from 6 leaves how many? 4 from 6 leaves how many?

7. How many birds are five birds and two birds? 5 and 2 are how many? 2 and 5 are how many?

8. If we take 2 birds from 7 birds, how many birds will be left? 2 from 7 leaves how many? 5 from 7 leaves how many?

LESSON VIII.

INCREASING AND DIMINISHING BY 3.

1. Three balls and three balls are how many balls?

2. If we take 3 marbles from 6 marbles, how many marbles will be left? 3 from 6 leaves how many?

3. How many pears are 4 pears and 3 pears? 4 and 3 are how many? 3 and 4 are how many?

4. 3 apples from 7 apples leaves how many apples? 3 from 7 leaves how many? 4 from 7 leaves how many?

5. Five cherries and three cherries are how many cherries? 3 units and 5 units are how many units?

6. If we take three plums from 8 plums, how many plums will be left? 3 units from 8 units leaves how many units? 5 from 8 leaves how many?

7. How many roses are 6 roses and 3 roses? 6 and 3 are how many? 3 and 6 are how many?

8. Three trees from nine trees leaves how many trees? 3 from 9 leaves how many? 6 from 9 leaves how many?

9. How many apples are seven apples and three apples? How many are 7 and 3? How many are 3 and 7?

10. If we take away 3 apples from 10 apples, how many apples will be left? 3 from 10 leaves how many? 7 from 10 leaves how many?

LESSON IX.

INCREASING AND DIMINISHING BY 4.

1. How many balls are 4 balls and 4 balls?

2. If we take away 4 marbles from 8 marbles, how many marbles will be left?

3. 5 sheep and 4 sheep are how many sheep? 5 and 4 are how many? 4 and 5 are how many?

4. If we take away 4 horses from 9 horses, how many horses will be left? 4 from 9 leaves how many? 5 from 9 leaves how many?

5. How many flowers are 6 flowers and 4 flowers? 6 and 4 are how many? How many are 4 and 6?

6. 4 roses from 10 roses leaves how many roses? 6 from 10 leaves how many? 4 from 10 leaves how many?

7. How many balls are 7 balls and 4 balls? 7 and 4 are how many? 4 and 7 are how many?

8. If we take 4 marbles from 11 marbles, how many marbles will be left? 4 from 11 leaves how many? 7 from 11 leaves how many?

9. Jane has eight pears, and Julia has four pears; how many pears have both? 8 and 4 are how many? 4 and 8 are how many? 4 and 5 and 2 are how many?

LESSON X.

INCREASING AND DIMINISHING BY 5.


1. How many sheep are 5 sheep and 5 sheep?

2. If we take 5 sheep from 10 sheep, how many sheep will be left? 5 from 10 leaves how many?


3. There are 5 pears on one branch and 6 pears on the other; how many pears on both branches?

4. If we take 5 pears from 11 pears, how many pears will be left? 6 from 11 leaves how many?


5. How many are 5 and 7? How many are 7 and 5?

6. If we take 5 lilies from 12 lilies, how many lilies will be left? 5 from 12 leaves how many? 7 from 12 leaves how many?


7. How many acorns are 5 acorns and 8 acorns? 5 and 8 are how many? How many are 8 and 5?

8. 5 from 13 leaves how many? 8 from 13 leaves how many?

9. Five and nine, how many? 5 from 14, how many?

LESSON XI.

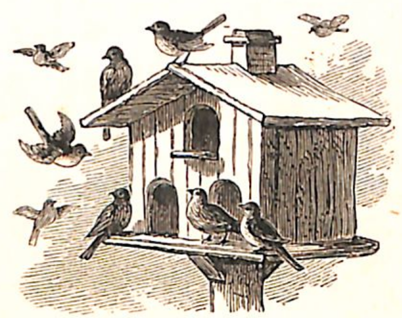
EXERCISES.


1. How many are nine cherries and three cherries? 9 and 3 are how many? 3 and 9 are how many?

2. If we take 9 balls from 11 balls, how many balls are left? 9 from 11 leaves how many? 2 from 11 leaves how many? 9 books from 11 books how many?

3. Here is a flock of swans; 4 are on land, and 5 on the water: how many in all? How many are five and four?

4. 4 from 9 leaves how many? 5 from 9 leaves how many?



5. How many birds are on the roof of the bird-house? How many are flying in the air? How many are on the shelf? How many are there in all? 2, 4 and 3 are how many? If the birds on the shelf fly away, how many will be left? 3 from 9 leaves how many?

LESSON XII.

EXERCISES.

1. How many boys are skating towards the right? How many towards the left? How many in all?

2. If five leave the ice, how many will be left skating. 6 from 11 leaves how many? 5 from 11, how many?

3. Here is a book-rack containing books; some are standing up, and some are lying down. How many are standing on the lower shelf? How many are lying on the lower shelf? How many books are there altogether on the lower shelf?

4. How many books are standing on the upper shelf? How many are lying down? How many books are there altogether on the upper shelf?

5. How many more are lying down on both shelves than there are standing?

LESSON XIII.

EXERCISES.

1. Two acorns and two acorns are how many acorns? How many lemons are 2 lemons and 2 lemons? How many acorns are 2 times 2 acorns? How many lemons are 2 times 2 lemons? How many are 2 times 2?

2. Two apples from four apples, leaves how many apples? 2 pears from 4 pears, leaves how many pears? How many times 2 apples are 4 apples? How many times 2 pears in 4 pears? 2 in 4 how many times?

3. How many sheep are 3 sheep and 3 sheep? How many sheep are 2 times 3 sheep? 3 times 2 sheep?

4. How many times 2 eggs, are there in 6 eggs? How many times 2 boys, in 6 boys? How many times 2 in 6? 3 times 2 are how many?

5. How many marbles are four marbles and four marbles? How many marbles are 2 times 4 marbles? How many are 2 times 4?

6. How many times 2 boats are there in 8 boats? How many times is 2 contained in 8?

7. If there are 2 bunches of acorns, and each bunch contains 5 acorns; how many acorns are there in both? How many are 2 times 5 acorns? How many are 2 times 5?

LESSON XIV.

WRITING HIGHER NUMBERS BY MEANS OF FIGURES.

Numbers from ninety-nine to one-thousand are written by three figures. The figure on the right, as we have already learned, stands for units, the second figure from the right stands for tens, the third figure stands for hundreds.

If there are no units, the figure on the right is 0.

If there are no tens, the second figure from the right is 0.

We will now write the numbers from one hundred to one hundred and twenty by means of figures:

101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120.

Two hundred is written by putting 2 in the place of hundreds, 0 in the place of tens, and 0 in the place of units; thus, 200.

Write two hundred and one; two hundred and two; two hundred and three; write two hundred and twelve.

Write three hundred; four hundred; five hundred; six hundred; seven hundred; eight hundred; nine hundred; two hundred and twenty-five.

Write nine hundred and nine; nine hundred and ninety nine; six hundred and four.

One thousand is written thus, 1000.

One thousand, one hundred, one ten, and one unit, as one number, are written thus, 1111; and the whole number is read one thousand one hundred and eleven.

Write in figures one thousand two hundred fifteen; one thousand and five; and one thousand and ten.

LESSON XV.

WRITING NUMBERS BY LETTERS.

We have learned two methods of writing numbers, one by *words*, and another by *figures*.

We will now learn a third method; the lessons in this book are numbered by this method.

In this method we use seven letters, I, V, X, L, C, D, M.

The following table shows how numbers are expressed by these seven letters.

TABLE.

1 . . .	I.	21 . . .	XXI.
2 . . .	II.	22 . . .	XXII.
3 . . .	III.	23 . . .	XXIII.
4 . . .	IV.	24 . . .	XXIV.
5 . . .	V.	25 . . .	XXV.
6 . . .	VI.	26 . . .	XXVI.
7 . . .	VII.	27 . . .	XXVII.
8 . . .	VIII.	28 . . .	XXVIII.
9 . . .	IX.	29 . . .	XXIX.
10 . . .	X.	30 . . .	XXX.
11 . . .	XI.	40 . . .	XL.
12 . . .	XII.	50 . . .	L.
13 . . .	XIII.	60 . . .	LX.
14 . . .	XIV.	70 . . .	LXX.
15 . . .	XV.	80 . . .	LXXX.
16 . . .	XVI.	90 . . .	XC.
17 . . .	XVII.	100 . . .	C.
18 . . .	XVIII.	500 . . .	D.
19 . . .	XIX.	1000 . . .	M.
20 . . .	XX.	1000000 . . .	$\overline{M}$.

NOTATION AND NUMERATION.

We have three methods of expressing numbers.

1. By words—called the **Word Method**; as one, two, three, four, etc.

2. By figures—called the **Arabic Method**; as 1, 2, 3, 4, etc.

3. By letters—called the **Roman Method**; as I, II, III, IV, etc.

Write all the numbers to fifteen, by each of the three methods.

The method of *writing* numbers is called **Notation.**

The method of *reading* written numbers is called **Numeration.**

RECAPITULATION AND DEFINITIONS.

1. A Unit is a single thing.

2. A Number is one or more things of the same kind.

3. A Figure is a character used to denote a number.

4. Notation is the method of writing numbers.

5. Numeration is the method of reading written numbers.

6. The Word Notation is the method of writing numbers by means of words.

7. The Arabic Notation is the method of writing numbers by means of figures.

8. The Roman Notation is the method of writing numbers by means of letters.

9. The ten figures taken separately are called **digits.** * The naught is also called **cipher** or **zero,** and when considered by itself, has *no value.* The other figures are called **significant,** because each has a value.

10. Arithmetic is the science of numbers, and the art of computing by them.

TEST QUESTIONS.

What is a unit? What is a number? Write five numbers; write seven numbers. How many units in each? In how many ways can we express numbers? What is the first method? What is it called? What is the second? What is it called? What is the third? What is it called? What is notation? What is numeration? What is Arithmetic?

ORDERS OF UNITS.

11. How many fingers and thumbs have you? Write the number by a word.

Will any *one* of the ten figures express this number?

What is the greatest number that one figure will express?

One book is a single thing—a unit. If we make a bundle of ten books this *bundle* is a single thing, and is, therefore, a unit. But these units are not alike. One unit is a *single book;* the other is a *single bundle* of ten books. The single book is called a *unit of the first order;* the single bundle of ten books is called a *unit of* the *second order.*

How many single books make the single bundle?

How many units of the first order in one unit of the second order?

If we make ten bundles with ten books in each bundle and then place the ten bundles together, making one large bundle, the large bundle will also be *one thing*—a unit. How many small bundles in the large bundle?

This last mentioned unit is a unit of the third order. How many units of the second order make one unit of the third order?

Write the figure one, and at the left of it write another figure one. The first one is a unit of the first order. The second one is a unit of the second order.

How many units of the first order make one unit of the second order?

Write another figure one at the left of the second. This last one expresses a unit of the third order.

How many units of the second order make one unit of the third order?

Write another figure one at the left of the last. This is a unit of the fourth order. How many units of the third order make one unit of the fourth order?

Now write, without assistance, a unit of the fifth order and a unit of the sixth order.

Write with one figure two units of the first order; two of the second; two of the third; two of the fourth; two of the fifth; and two of the sixth.

NOTE.—*Always remember that one order of figures occupies but one place, and the largest number of any one order is nine.*

Write in figures, as one number, three units of the first order, four units of the second order, two units of the third order, five units of the fourth order, one unit of the fifth order, and nine units of the sixth order.

These numbers are *integers*.

12. An Integer is a whole number.

13. Units of the First Order either stand alone, or occupy the right-hand place.

14. Units of the Second Order occupy the second place from the right.

15. Units of the Third Order occupy the third place.

You may now tell what place units of the fourth order occupy; units of the fifth order, sixth order, seventh order, eighth order, ninth order, tenth order.

Units of the second order may be expressed without a unit of the first order, by putting a cipher in the place of the unit of the first order. Thus, 10.

The orders of units are *indicated* by the *relative positions of the figures.*

Units of any order may be written without expressing the units of other orders by putting ciphers in the place of the other units. Thus, two units of the third order are written 200; two units of the first order and four units of the fifth order are written thus, 40002, ciphers taking the place of the absent units.

Write two tens. What number have you written?

Write three tens, and at the right of it two units. What number have you now?

Write three units of the fourth order, and in the same line one unit of the third order, five units of the second order, and nine units of the first. What number do they express?

Units of the First Order express *single things*, and are called simply *units*.

Units of the Second Order express *collections of ten single things*, and are therefore called *tens*.

Units of the Third Order express *collections of ten tens*, or *one hundred*, and are therefore called *hundreds*.

Units of the Fourth Order express collections of *ten hundreds* or one *thousand*, and are therefore called *thousands*, as shown in the following

NUMERATION TABLE.

Hundreds of Trillions.	Tens of Trillions.	Trillions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
2	1	3	5	4	6	9	7	8	0	7	8	4	5	3

NOTE TO TEACHERS.—Minds are not all formed in the same mould. The teacher will find an unlimited variety, and must be ready to vary his methods of teaching accordingly.

SIMPLE AND LOCAL VALUES.

Another method of presenting this subject is the following. It is inserted to meet the wants of those who fail to comprehend the first method.

16. Figures have two values, called a **Simple Value** and a **Local Value.**

The **Simple Value of a Figure** is its value when standing alone, or when used as the right hand figure of a number.

The **Local Value of a Figure** is its value arising from the place in which it stands. When 2 stands alone or at the right hand, it denotes 2 units; when it stands in the second place from the right, it denotes 2 *tens*, as in 24; when it stands in the third place from the right, as in 234, it denotes 2 *hundreds*. The local values of figures increase from the right to the left by a **scale of tens.**

TEST QUESTIONS.

What do units of the first order express? Units of the second order? Third? Fourth? How many values have figures? What is the *simple* value of a figure? What is the *local* value? Write 2 so as to show its simple value. Write 2 so as to denote 2 tens; to denote 2 hundreds. How do the local values increase?

Places of *figures* and *orders of units* are counted from *right* to *left*, but numbers are read from *left* to *right.*

EXAMPLES IN NOTATION AND NUMERATION.

17. *1.* Numbers from one to nine inclusive are collections of simple units, and are expressed by a single figure.

2. Numbers from ten to ninety-nine inclusive are composed of tens and units. Thus, twenty-seven is composed of 2 tens and 7 units; forty-eight is composed of 4 tens and 8 units.

Write the following numbers by means of figures:

1. Twenty-five.	*6.* Thirty-two.	*11.* Forty-two.
2. Forty-one.	*7.* Fifty-eight.	*12.* Ninety-four.
3. Fifty-seven.	*8.* Nineteen.	*13.* Eighty-two.
4. Eighty.	*9.* Twelve.	*14.* Sixty-five.
5. Ninety-one.	*10.* Seventy-six.	*15.* Ninety-nine.

Read the following numbers:

84.	62.	94.	38.	27.	33.	76.
79.	**41.**	**81.**	**54.**	**48.**	**87.**	**78.**

3. Numbers from one hundred to nine hundred and ninety-nine inclusive are composed of hundreds, tens, and units. Thus, the number four hundred and sixty-five is composed of 4 *hundreds*, 6 *tens*, and 5 *units;* two hundred and three is composed of 2 *hundreds*, 0 *tens*, and 3 *units.*

EXAMPLES.

Write the following numbers by means of figures:

1. Two hundred and sixty-five.
2. Three hundred and ninety.
3. Seven hundred and eight.
4. Eight hundred and fifty-seven.
5. Nine hundred and eighty.
6. Four hundred and thirty-two.
7. Two hundred and six.
8. One hundred and ninety-nine.
9. Five hundred and seventy-three.
10. Six hundred and sixty-six.

4. To read a number of three figures, we name the number of hundreds and then read the tens and units, as though they were by themselves. Thus, 512 is read, five hundred and twelve; 874 is read, eight hundred and seventy-four; 209 is read, two hundred and nine.

Read the following numbers:

1.	713.	*7.*	495.	*13.*	642.	*19.*	384.
2.	806.	*8.*	888.	*14.*	404.	*20.*	763.
3.	200.	*9.*	232.	*15.*	546.	*21.*	914.
4.	817.	*10.*	450.	*16.*	634.	*22.*	571.
5.	728.	*11.*	527.	*17.*	978.	*23.*	994.
6.	827.	*12.*	932.	*18.*	769.	*24.*	999.

18. Numbers are written by putting each order in its own place, and if any order is not mentioned, naught must occupy its place.

Write the following numbers by means of figures:

1. Six thousand four hundred and twenty-one.
2. One thousand three hundred and five.
3. Two thousand and ninety-six.
4. Eight thousand and one.
5. Four thousand and nine hundred.
6. Sixty-one thousand three hundred and seven.
7. Twenty-three thousand seven hundred and five.
8. Two hundred and fifty-nine thousand.
9. Three hundred, forty-eight thousand and thirteen.

Read the following numbers, and name the number of units in each order:

(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)
1423	2010	23705	67832	258013
(*6.*)	(*7.*)	(*8.*)	(*9.*)	(*10.*)
2567	1365	61307	57243	700230

PERIODS OF FIGURES.

19. Numbers containing more than three figures are separated into periods of three figures each, beginning at the right. The left-hand period may contain less than three figures.

The first period, counting from the right, is called the *period of units*, the second is called the *period of thousands;* the third is the *period of millions*, and so on, as shown in the following table:

Periods . . .	trillions,			billions,			millions,			thousands,			units.		
	hundreds of	tens of	units of	hundreds of	tens of	units of	hundreds of	tens of	units of	hundreds of	tens of	units of	hundreds of	tens of	units of
Number . . .	2	3	1,	8	7	6,	4	1	5,	3	0	0,	2	1	0.

The number written above is read, 231 *trillions*, 876 *billions*, 415 *millions*, 300 *thousands*, 210.

The table may be extended at pleasure; the units of the succeeding periods are *quadrillions*, *quintillions*, *sextillions*, etc.

Every period except the left-hand one must be *complete*, that is, it must contain three digits, but one or all these digits may be ciphers.

Divide 14674268436173 into periods. Tell how many figures in each period, and read the number.

DEFINITION.

20. A **Rule** is a brief direction for performing work.

RULE FOR NOTATION.

Begin at the left and write the figures of each period in their proper order, filling all vacant places with ciphers.

RULE FOR NUMERATION.

I. Begin at the right and separate the number into periods of three figures each, until you reach the left-hand period, which may have one, two, or three figures.

II. Begin at the left and read each period as if it stood alone, naming each period as you read its last figure.

Write, point off, and read the following numbers:

2345	421121	89587346
14800	103043	6129456013
21576	7271856	9078645327
743209	234517	12769853412
825364	100200	8742186548214

TEST QUESTIONS.

How many periods in the last number? Name the periods in this number. How many figures in each period? Give the rule for notation. Give the rule for numeration. Which period may have less than three figures?

CLASSIFICATION OF NUMBERS.

Count five. In this manner of counting, you mention the numbers without a thought of any object. Numbers used in this way are called **abstract numbers.**

Count the number of scholars in this class; the number of maps on the wall; the number of books on my desk; the number of panes of glass in the window. Numbers used in connection with the objects counted, indicating the *number* of *objects*, are called **denominate** or **concrete.**

DEFINITIONS.

21. An **Abstract Number** is one whose unit is not named; as *three, five, seven.*

22. A **Denominate** or **Concrete Number** is one whose unit is named; as *three girls, five pounds seven pennies, eight pencils.*

Name five abstract numbers. Name five denominate numbers. Write four abstract numbers. Write five denominate numbers. What is an abstract number? What is a denominate number

23. An **Integer** or **Integral Number** is one which expresses one or more entire things.

24. The **Unit** of any number is one of the collection which constitutes the number.

25. Similar or **Like Numbers** are those that have the same kind of unit; as *eight days* and *ten days, two yards seven feet,* and *six yards eleven feet.*

26. Unlike Numbers are those that have different kinds of units; as *eight horses* and *five cows, six pencils and four knives, two feet* and *three days.*

REVIEW QUESTIONS.

What is a unit? Write a unit. What is a number? Write two numbers. What is arithmetic? What is notation? What is numeration? Name the three methods of notation. The Arabic notation employs how many figures? Write them. What are these figures called taken separately? What is an abstract number? Give two examples. What is a concrete, or denominate number? Give three examples. What is an integer? What are like numbers? Give two examples. What are unlike numbers? Give two examples. What are the first four orders of units? Write four units of the third order. How many units of any order make one unit of the next higher order? What is the greatest number that can be expressed by one figure? What is the greatest number that can be expressed by two figures? What by three figures? When there are four figures in a number, of what orders is it composed? Give an example and name the *order* of each figure. What do units of the first order denote? What do units of the second order denote? What do units of the third order denote? What is the simple value of a figure? What is the local value of a figure? Give all the general principles of notation and numeration. Give the rule for notation. Give the rule for numeration. How are numbers expressed in the Roman notation? What are figures? Name all the orders to trillions, beginning with units. Name the first four periods. Why is the second order called tens? Why third called hundreds? Why fourth called thousands?

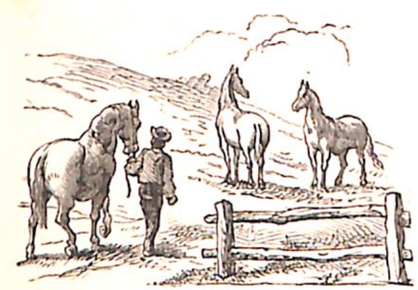
FUNDAMENTAL OPERATIONS.

ADDITION.

1. Mary has a *doll,* and her mother gives her another *doll;* how many *dolls* has she then?

Write the number of dolls that Mary has.

How many are 1 and 1? How many are 1 and 1 and 1? How many are 1 and 1 and 1 and 1?



2. If a farmer has 2 *horses* in the pasture and his neighbor puts in 1 more; how many *horses* will there be in the pasture? How many are 2 and 1? 3 and 1? 4 and 1? 5 and 1? 6 and 1? 7 and 1? 8 and 1? 9 and 1? 10 and 1? 12 and 1? 17 and 1?

Write the number of horses in the picture.

3. How many *dimes* are 3 *dimes* and 2 *dimes?* How many are 2, and 1, and 1? How many are 3, 1, and 1? How many are 4, 1, and 1? How many are 5, 1, and 1? How many are 6, 1, and 1?

4. How many *cents* are 4 *cents*, 2 *cents*, and 1 *cent?* How many are 3, 2, and 1? How many are 2, 3, and 1?

5. How many *men* are three *men*, two *men*, and two *men?* How many are 3, 2 and 2?

5. How many *marbles* are 4 *marbles*, 3 *marbles*, and 2 *marbles?* How many are 3, 4 and 2?

6. The word *bridge* has 6 *letters*, the word *man* has 3 *letters*, and the word *up* has 2 *letters;* how many *letters* in the three words? How many in the first two words?

The operation of finding how many *dolls* Mary has, how many horses there are in the pasture, etc., is called *Addition*, and the number thus found is called the *Sum* or *Amount*.

DEFINITIONS.

27. The sum of two or more numbers is a number which contains as many units as all the numbers taken together.

28. Addition is the operation of finding the sum of two or more numbers.

The numbers to be added must be similar, that is, they must have the same unit. Three days and two days can be added, because they have the same unit, one day; but three days and two yards cannot be added, because they have not the same unit.

What is the sum or amount of two or more numbers? What is addition? What are similar or like numbers?

SIGNS.

In the examples given above, the word *and* is used to denote the *addition;* we generally denote it by this sign +, which is called **plus**, and when it is used between numbers it shows that they are to be added; thus 6+3+2 are 11, means that the sum of *six* and *three* and *two* is equal to *eleven*.

In place of the word *are*, the sign = is used. It is called the *sign* of *equality*, and is read *equals* or *equal to;* thus, $6 + 4 + 8 = 18$, is read, *six* plus *four* plus *eight* equals *eighteen*.

What is the sign of addition? Make it. What does it denote? What is the sign of equality? Make it.

29. The sign of equality placed between numbers or combinations of numbers, shows that those at the left hand are equal to those at the right.

The entire expression is called an **equation**; thus, $6 + 3 = 9$, $7 - 2 = 5$, $8 \times 3 \div 2 = 14 - 6 + 4$, are equations.

30. An Arithmetical Equation is the expression of equality between numbers or combinations of numbers.

What is an equation? Write an equation.

EXERCISES FOR ORAL WORK.

1+1 = ?	2+2 = ?	3+3 = ?	4+4 = ?
2+1 = ?	3+2 = ?	4+3 = ?	5+4 = ?
3+1 = ?	4+2 = ?	5+3 = ?	6+4 = ?
4+1 = ?	5+2 = ?	6+3 = ?	7+4 = ?
6+1 = ?	6+2 = ?	7+3 = ?	8+4 = ?

31. Make and learn the following

ADDITION TABLE.

2+ 0= 2	3+ 0= 3	4+ 0= 4	5+ 0= 5
2+ 1= 3	3+ 1= 4	4+ 1= 5	5+ 1= 6
2+ 2= 4	3+ 2= 5	4+ 2= 6	5+ 2= 7
2+ 3= 5	3+ 3= 6	4+ 3= 7	5+ 3= 8
2+ 4= 6	3+ 4= 7	4+ 4= 8	5+ 4= 9
2+ 5= 7	3+ 5= 8	4+ 5= 9	5+ 5=10
2+ 6= 8	3+ 6= 9	4+ 6=10	5+ 6=11
2+ 7= 9	3+ 7=10	4+ 7=11	5+ 7=12
2+ 8=10	3+ 8=11	4+ 8=12	5+ 8=13
2+ 9=11	3+ 9=12	4+ 9=13	5+ 9=14
2+10=12	3+10=13	4+10=14	5+10=15
6+ 0= 6	7+ 0= 7	8+ 0= 8	9+ 0= 9
6+ 1= 7	7+ 1= 8	8+ 1= 9	9+ 1=10
6+ 2= 8	7+ 2= 9	8+ 2=10	9+ 2=11
6+ 3= 9	7+ 3=10	8+ 3=11	9+ 3=12
6+ 4=10	7+ 4=11	8+ 4=12	9+ 4=13
6+ 5=11	7+ 5=12	8+ 5=13	9+ 5=14
6+ 6=12	7+ 6=13	8+ 6=14	9+ 6=15
6+ 7=13	7+ 7=14	8+ 7=15	9+ 7=16
6+ 8=14	7+ 8=15	8+ 8=16	9+ 8=17
6+ 9=15	7+ 9=16	8+ 9=17	9+ 9=18
6+10=16	7+10=17	8+10=18	9+10=19

NOTE.—The teacher will be amply repaid for thorough drill in all possible combinations of the digits. The pupil should be thoroughly master of the table. Frequent exercises are required to secure this and to break up the habit of counting, which is fatal to rapidity in addition.

EXAMPLES FOR ORAL WORK.

1. 2+7 = ?	*5.* 9+5 = ?	*9.* 9+2 = ?
2. 4+9 = ?	*6.* 8+4 = ?	*10.* 7+8 = ?
3. 8+6 = ?	*7.* 8+9 = ?	*11.* 6+9 = ?
4. 3+5 = ?	*8.* 3+8 = ?	*12.* 8+7 = ?

13. 3+4+2+5 = ? *16.* 9+3+2+4 = ?
14. 4+5+3+4 = ? *17.* 1+3+4+3+2 = ?
15. 7+3+2+5 = ? *18.* 3+2+3+4+2+3 = ?

19. 4+3+2+3+1+6 = ? 3+8+5+4+6 = ?
20. 5+2+3+1+2+4+5 = ? 7+9+8+5 = ?
21. 8+5+4+2+3+1+7 = ? 9+3+6+4 = ?
22. 9+6+3+4+6+2+1 = ? 4+9+8+7 = ?

NOTE.—The signs of interrogation in these and all similar examples indicate that the second members are to be supplied by the pupil.

23. Count by twos from 2 to 50; thus, 2, 4, 6, 8, 10, etc.
24. Count by threes from 3 to 99; from 27 to 120.
25. Count by fours from 0 to 100; from 105 to 161.
26. Count by fives from 1 to 106; from 200 to 250.
27. Count by sixes from 0 to 108; from 212 to 242.
28. Count by sevens from 0 to 98; from 100 to 135.
29. Count by eights from 0 to 104; from 150 to 174.
30. Count by threes from 4 to 46; from 50 to 77.
31. Count by fives from 0 to 100; from 200 to 250.
32. Count by nines from 0 to 99; from 100 to 154.

32. Instead of writing numbers in a horizontal line with the sign between them, it is more convenient to write them in columns without any sign, the sum being written beneath. In the following examples, add from the bottom upward.

EXAMPLES FOR WRITTEN WORK.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)
4	6	8	8	9	8	6	1	5
5	8	3	8	3	4	7	2	9
7	2	8	4	9	3	3	5	4
3	9	7	7	5	5	8	9	7
19								

In adding, name only the results of each addition: thus, Example (1), *three, ten, fifteen, nineteen*; Example (2), *nine, eleven, nineteen, twenty-five.*

Prove the work by adding from the top downward; if the same sum is obtained, the work is thought to be right.

Add and prove the following examples:

(10.)	(11.)	(12.)	(13.)	(14.)	(15.)	(16.)	(17.)	(18.)
7	9	7	8	5	8	5	7	8
8	8	3	4	7	1	5	6	3
1	4	6	9	4	6	3	5	2
5	3	5	3	3	9	2	9	5
4	9	2	2	9	4	9	4	7

(19.)	(20.)	(21.)	(22.)	(23.)	(24.)	(25.)	(26.)	(27.)
8	8	4	4	1	9	4	6	8
4	2	6	5	3	1	5	2	9
3	8	5	9	7	8	7	6	4
6	3	3	2	2	6	2	3	2
2	7	8	1	4	3	8	4	1
1	9	9	3	9	2	6	7	8

28. Find the sum of 3, 4, 8, 9, 7, 4, and 5.

29. Find the sum of 6, 3, 2, 1, 8, 9, 4, 5, and 9.

30. Find the sum of 9, 8, 6, 7, 2, 5, 8, 7, and 17.

31. Find the sum of 8, 7, 9, 6, 3, 5, 1, 4 and 5.

Simple numbers may be added by the following

RULE.

I. Write the numbers so that units of the same order shall stand in the same column.

II. Begin at the right, add each column, and write the sum, if less than ten, under the column.

III. When the sum of any column exceeds nine, set down the right-hand figure of the sum under the column, and add the number indicated by the left-hand figure or figures to the next column.

IV. Continue this operation till all the columns have been added; write the entire sum of the last column.

Proof

PROOF.—*Add the numbers from the top downward; if the result is the same as the first sum, the work is presumed to be right.*

EXAMPLES.

	(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
	24	30	45	81	16	42
	32	23	33	72	12	33
	51	64	61	63	27	21
	70	72	70	54	38	48
Sum,	177					

(7.)	(8.)	(9.)	(10.)	(11.)	(12.)
36	18	84	44	88	86
47	41	62	37	66	60
58	82	31	18	23	45
79	61	14	29	32	57

(13.)	(14.)	(15.)	(16.)
213	127 *days*	236 *quarts*	812
416	213 *days*	72 *quarts*	174
729	418 *days*	801 *quarts*	702
Sum, 1358	758 *days*		

In the last four examples, which answers are abstract? Which are concrete?

Can 127 *days* be added to 236 *quarts?* Why not?

EXAMPLES.

17. Find the sum of 125, 718, 64, 376, and 715.

18. Find the sum of 73 *years*, 172 *years*, 60 *years*, 812 *years*, 43 *years*, and 197 *years*.

19. Add 345 *quarts*, 117 *quarts*, 123 *quarts*, 885 *quarts*, 64 *quarts*, and 543 *quarts*.

20. Add 2,135 *pounds*, 8,126 *pounds*, 3,152 *pounds*, 8,176 *pounds*, and 364 *pounds*.

21. Find the sum of 77, 213, 315, 421, and 607.

Add the following groups of numbers:

22. 818, 328, 40, 671, 364, 484, and 793.

23. 15, 812, 75, 717, 645, 720, and 347.

24. 412 *days*, 817 *days*, 516 *days*, and 893 *days*.

ABBREVIATIONS.—In what follows, *ft.* stands for *feet; yds.* for *yards; lbs.* for *pounds; in.* for *inches; qts.* for *quarts;* and the sign $ placed before a number stands for *dollars*.

When dollars and cents are expressed, we first write the sign, then the number of dollars, then a point or period, and then the number of cents: thus, $25.75, read twenty-five dollars and seventy-five cents. If the number of cents to be written is less than ten, a cipher must be put in the tens place; thus, $16.05, read sixteen dollars and five cents. If cents alone are to be written, we first make the sign of dollars, then a 0, then the point, and then write the number of cents; thus, $0.16, read 16 cents. When dollars, cents, and mills are to be expressed, write the dollars and cents as above, and the mills at the right of the cents. Seven dollars, twenty-five cents and eight mills are written $7.258.

EXAMPLES.

Find the sum of

25. 512 *ft.*, 893 *ft.*, 911 *ft.*, 745 *ft.*, and 14 *ft.*

26. 482 *yds.*, 886 *yds.*, 924 *yds.*, 87 *yds.*, and 994 *yds.*

27. 864 *lbs.*, 342 *lbs.*, 182 *lbs.*, 94 *lbs.*, and 14 *lbs.*

28. 667 *in.*, 843 *in.*, 918 *in.*, 445 *in.*, and 887 *in.*

29. $818, $435, $88, $413, $867, $983, and $71.

30. 2,314, 12,107, 210,026, 78,784, and 68,547.

31. $2,308, $12,125, $41,410, and $18,876.

32. 1,280 *yds.*, 71,413 *yds.*, 47,489 *yds.*, and 9,297 *yds.*

33. 1,762 *lbs.*, 4,389 *lbs.*, 120,000 *lbs.*, and 172,794 *lbs.*

34. 842 *ft.*, 8,848 *ft.*, 27,796 *ft.*, 152,407 *ft.*, and 18 *ft.*

35. $64, $640, $6,832, $9,040, $118,920, and $8,734.

36. 614 *in.*, 3,260 *in.*, 89,705 *in.*, 8,884 *in.*, and 286 *in.*

DEFINITIONS.

33. A **Problem** is a question requiring a solution.

34. A **Solution** is the operation of finding the required answer.

PRACTICAL PROBLEMS FOR ORAL WORK.

35. *1.* Jane's father gave her 6 peaches and her mother gave her 4 more; how many peaches did both give her?

SOLUTION. 6 peaches + 4 peaches = 10 peaches. *Ans.*

2. Charles gave 10 cents for a Faber pencil No. 2, 5 cents for an Eagle pencil No. 2, and 8 cents for a Stoddard pencil; what did the three pencils cost him?

3. The head of a fish caught in Newark Bay was 6 inches long, its body 21 inches long, and its tail 7 inches long; how long was the fish?

4. Count by 7's from 2 to 79; from 5 to 96; from 4 to 102; from 9 to 51; from 11 to 74.

5. George solved 19 problems in the morning and 8 in the evening; how many did he solve in all?

For Written Work.

6. A grocer has 3 hogsheads of sugar, of which the first weighs 957 *lbs.*, the second 1,023 *lbs.*, and the third 1,179 *lbs.*; what is the weight of them all?

Explanation. — The weight of the whole is equal to the sum of the weights of all the parts. Hence, we set down the separate weights and add them.

ILLUSTRATION.

	957 *lbs.*
	1023 *lbs.*
	1179 *lbs.*
Ans.	3159 *lbs.*

7. A merchant bought 4 pieces of cloth for $129, 6 pieces of silk for $312, and 97 pieces of muslin for $873; what did he pay for them all? *Ans.* $1,314.

8. A gentleman bought a pair of horses for $650, a set of harness for $190, and a carriage for $955; what did they all cost?

9. A merchant bought a horse for 112 dollars; after keeping him a short time, he sold him, and gained 25 dollars; how much did he receive for the horse?

10. The mail route from Albany to New York is 144 miles, from New York to Philadelphia 90 miles, from Philadelphia to Baltimore 98 miles, and from Baltimore to Washington City 38 miles; what is the distance from Albany to Washington?

FOR ORAL WORK.

36. *1.* A man bought a firkin of butter for $9, a keg of syrup for $6, and 5 bushels of wheat for $7; how much did he give for the whole?

2. A boy gave to one of his companions 7 apples, to another 6, and to a third 8; how many apples did he give away? $7+6+8=?$

3. A farmer bought a cow for $30, a sheep for $20, and a calf for $10; how much did he give for the whole?

4. In a young peach orchard Jane found 27 peaches on one tree, on another 10, on another 8, and on another 5; how many peaches did she find?

5. A lady bought a muff for $25, a boa for $15, and a pair of gaiters for $10; how much did she pay for the whole? $25 + $15 + $10 = ?

FOR WRITTEN WORK.

1. A grocer sold 289 pounds of sugar for $28, ten barrels of flour for $108, and a quantity of pork for $879; what did he get for the whole?

2. A person pays $950 for a lot of ground, on which he builds a house costing $5,430, a barn costing $986, and then sells the whole so as to gain $914; what was his selling price?

3. A farmer raises 673 bushels of wheat, 1,489 bushels of corn, 67 bushels of barley, and 1,682 bushels of oats; how many bushels of grain does he raise in all?

4. A farmer sells his stock of cattle as follows: for his oxen he gets $883, for his cows $1,279, for his calves $413, and for his horses $980; what does he get for them all?

5. A gentleman builds a house: his lot costs him $1,254, the carpenter work costs $4,320, the masonry $2,110, the painting and papering $1,187, and the miscellaneous expenses amount to $1,277; what is the cost of the whole?

6. The distance from Boston to Springfield is 99 miles, from Springfield to Albany 102 miles, from Albany to Rochester 226 miles, from Rochester to Buffalo 65 miles, and from Buffalo to Chicago 518 miles; how far is it from Boston to Chicago by this route?

7. A manufacturer paid $8,820 for rent, $17,780 for material, $47,885 for labor, and then sold his goods so as to clear $11,827; what was the amount of his sales?

8. A speculator bought a house and lot for 1,964 dollars, expended 384 dollars in repairing and refitting the property, paid taxes and insurance amounting to 56 dollars, and then sold them so as to gain 396 dollars; what did he get for the property?

TEST QUESTIONS.

What is addition? What is the answer in addition called? What is the sign of addition? What does it mean? Make the sign of equality. Why is it called sign of equality? Write an example in which there is the sign of equality, and show how it is used. Give the rule for writing numbers in addition. Give the rule for adding and writing the results. How do you prove addition? What is an equation? What are the members of an equation? What numbers can be added together? Make the sign for dollars. How many orders of units or places do cents occupy? What sign is used between dollars and cents? How are dollars, cents, and mills written for adding? How many places do cents and mills occupy?

SUBTRACTION.

1. One of the boys in the picture has 4 *apples*, the other boy has 3 *apples*; how many more *apples* has the first boy than the second?

2. One of the girls has 4 *roses*, the other has 2 *roses*; how many more *roses* has one girl than the other?

3. On one side of the walk there are 5 *trees*, on the other side 2 *trees*; how many more *trees* on one side than on the other?

4. On one side of the house you can see 6 *windows*, on the other side 2 *windows*; how many more can you see on one side than on the other?

5. A man had 12 *cows*, and sold 3 of them; how many had he left?

In these five examples we are required to find *how much greater one number is than another.* The number thus found is the **Difference** *between the two numbers*, and the process of finding it is called **Subtraction**. The difference is also called a **Remainder**.

DEFINITIONS.

37. The **Difference**, or **Remainder**, is a number which shows how much greater one of two numbers is than the other.

38. Subtraction is the operation of finding the difference between two numbers.

In these examples, and in all examples of subtraction in Arithmetic, the greater number is called the **Minuend**. The less number is called the **Subtrahend**.

39. The **Minuend** is one of two numbers from which the other is to be subtracted.

40. The **Subtrahend** is the number to be subtracted.

What is meant by the difference between two numbers? What is subtraction? What is the minuend? What is the subtrahend? In each of the following examples tell which is the minuend and which the subtrahend.

Read and work the following

EXAMPLES.

1. 5 less 4 = 1.
2. 7 less 6 = how many?
3. 4 less 1 = how many?
4. 5 less 3 = how many?
5. 4 less 3 = how many?
6. 6 less 1 = how many?
7. 6 less 4 = how many?
8. 5 less 1 = how many?
9. 4 less 2 = how many?
10. 3 less 1 = how many?
11. 8 less 3 are how many? 8 less 4? 8 less 5?

Instead of the word *less* between two numbers whose difference is required, this sign — is used. It is called the **Minus Sign**, or **Sign of Subtraction**.

41. Minus denotes *less*, and when placed between two numbers it shows that the second is to be subtracted from the first. Thus, 5 — 3 shows that 3 is to be taken from 5.

42. The **Parenthesis**, ( ), is used to show that the expression enclosed by it is to be treated as a single number. Thus, 8 — (3 + 2) shows that the sum of 3 and 2 is to be subtracted from 8.

Read and work the following

EXAMPLES.

1. 6—1 = 5.	*7.* 5—3 = ?	*13.* 4—2 = ?
2. 4—1 = ?	*8.* 4—3 = ?	*14.* 6—3 = ?
3. 3—2 = ?	*9.* 2—1 = ?	*15.* 5—1 = ?
4. 5—2 = ?	*10.* 6—2 = ?	*16.* 6—4 = ?
5. 3—1 = ?	*11.* 5—4 = ?	*17.* 6—5 = ?
6. 8—4 = ?	*12.* 7—2 = ?	*18.* 9—6 = ?

19. 10—3 = how many? 10—5? 10—7? 10—8?
20. 12—1 = how many? 12—2? 12—3? 12—6?
21. How many are 13—1? 13—2? 13—3? 13—4?
22. How many are 14—1? 14—2? 14—3? 14—4?

Write the sign of subtraction. What is it called? What does it mean? When used between two numbers what does it show?

Read 8 — 3 = 5, and tell which is the minuend, which the subtrahend, and which the remainder?

Can you always tell, if the sign of subtraction is used, which is the minuend, and which is the subtrahend? How?

43. Write and learn the following

SUBTRACTION TABLE.

1 from	2 from	3 from	4 from	5 from
2 leaves 1	3 leaves 1	4 leaves 1	5 leaves 1	6 leaves 1
3 " 2	4 " 2	5 " 2	6 " 2	7 " 2
4 " 3	5 " 3	6 " 3	7 " 3	8 " 3
5 " 4	6 " 4	7 " 4	8 " 4	9 " 4
6 " 5	7 " 5	8 " 5	9 " 5	10 " 5
7 " 6	8 " 6	9 " 6	10 " 6	11 " 6
8 " 7	9 " 7	10 " 7	11 " 7	12 " 7
9 " 8	10 " 8	11 " 8	12 " 8	13 " 8
10 " 9	11 " 9	12 " 9	13 " 9	14 " 9
6 from	**7 from**	**8 from**	**9 from**	**10 from**
7 leaves 1	8 leaves 1	9 leaves 1	10 leaves 1	11 leaves 1
8 " 2	9 " 2	10 " 2	11 " 2	12 " 2
9 " 3	10 " 3	11 " 3	12 " 3	13 " 3
10 " 4	11 " 4	12 " 4	13 " 4	14 " 4
11 " 5	12 " 5	13 " 5	14 " 5	15 " 5
12 " 6	13 " 6	14 " 6	15 " 6	16 " 6
13 " 7	14 " 7	15 " 7	16 " 7	17 " 7
14 " 8	15 " 8	16 " 8	17 " 8	18 " 8
15 " 9	16 " 9	17 " 9	18 " 9	19 " 9

NOTE.—This table should be repeated until the scholar becomes familiar with it. Change the form thus: 2−1=1, etc. Again, change thus: What number taken from 2 leaves 1? etc.

The minuend, the subtrahend, and the remainder must be *similar*, or *like numbers*.

EXAMPLES.

ORAL WORK.

1. $7 - 4 = ?$
2. $8 - 3 = ?$
3. $9 - 6 = ?$
4. $12 - 9 = ?$
5. $11 - 4 = ?$
6. $6 - ? = 2.$
7. $5 - ? = 4.$
8. $10 - ? = 3.$
9. $12 - ? = 5.$
10. $13 - ? = 8.$
11. $12 - 3 = ?$
12. $14 - 5 = ?$
13. $15 - 6 = ?$
14. $16 - 7 = ?$
15. $15 - 8 = ?$
16. $15 - ? = 10.$
17. $13 - ? = 4.$
18. $14 - ? = 8.$
19. $12 - ? = 5.$
20. $15 - ? = 9.$

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21. $5 + 6 - 3 = ?$
22. $7 + 8 - 4 = ?$
23. $9 + 3 - 5 = ?$
24. $12+2+3-4 = ?$
25. $9+7+6-5 = ?$
26. $20 - (2 + 4) = ?$
27. $15 - (3 + 5) = ?$
28. $18-(1+2+3) = ?$
29. $20 - (3+4+5) = ?$
30. $19 - (7+6+1) = ?$
31. $12 - 2 - 2 - 2 = ?$
32. $15 - 3 - 2 - 4 = ?$
33. $20 - 2 - 5 - 3 = ?$
34. $18 - 5 - 4 - 3 = ?$
35. $20 - 6 - 5 - 3 = ?$
36. $18 + 3 - (6 - 3) = ?$

37. 20 diminished by 5 = ? 15 diminished by 5 ? 10 diminished by 5 ? 5 diminished by 5 ?

38. In the same manner name the numbers from 52 to 0, diminishing each by 2 ; from 78 to 52 ; from 100 to 78.

39. What are the numbers from 86 to 2, when each is diminished by 6 ? from 122 to 8 ?

44. Instead of writing numbers for subtraction in a horizontal line with minus between, it is generally more

3

convenient to write the subtrahend under the minuend, placing the remainder beneath; thus,

12	*Minuend.*
7	*Subtrahend.*
5	*Remainder.*

In this manner work the following

EXAMPLES.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)	(10.)
16	18	12	10	9	8	12	20	16	21
9	4	5	7	4	5	3	9	7	5
7									

To *prove the work*, add the remainder to the subtrahend, and if the sum is equal to the minuend the work is presumed to be right.

Perform and prove the following

EXAMPLES.

	(11.)	(12.)	(13.)	(14.)	(15.)	(16.)	(17.)	(18.)	(19.)
From	9	7	11	15	8	13	17	10	14
Subtract	2	3	7	6	4	5	9	6	8

	(20.)	(21.)	(22.)	(23.)	(24.)	(25.)	(26.)
From	14 *yds.*	12 *lbs.*	19 *ft.*	$13	16 *in.*	10 *lbs.*	15 *qts.*
Subtract	9 *yds.*	6 *lbs.*	10 *ft.*	$4	7 *in.*	3 *lbs.*	9 *qts.*

	(27.)	(28.)	(29.)	(30.)	(31.)	(32.)	(33.)	(34.)
From	8	10 *lbs.*	9 *yds.*	12 *ft.*	14 *in.*	17	$15	33 *days.*
Take	4	6 *lbs.*	2 *yds.*	4 *ft.*	8 *in.*	8	$9	21 *days.*

Can 9 *pounds* be subtracted from 12 *yards*? Why not?

Exercises for Written Work.

45. *When the figures of the subtrahend are equal to or less than the corresponding figures of the minuend.*

ILLUSTRATION.	
From	795
Subtract	532
Remainder,	263

Explanation.—We write the subtrahend under the minuend so that units stand under units, tens under tens, and hundreds under hundreds. We begin at the right and subtract 2 *units* from 5 *units* and write the remainder, 3 *units*, beneath. We then subtract 3 *tens* from 9 *tens* and write the result, 6 *tens*, in the column of tens. Then we subtract 5 *hundreds* from 7 *hundreds*, and write the difference in the column of hundreds.

Proof.

Add the subtrahend and remainder, and if the sum equals the minuend the work is right.

532	Subtrahend.
263	Remainder.
795	= the Minuend.

Second Method of Proof.

Subtract the remainder from the minuend, and if the result equals the subtrahend the work is right.

795	Minuend.
263	Remainder.
532	Subtrahend.

In the same manner work and prove the following

EXAMPLES.

	(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
From	87	76	95	69	82	98
Subtract	34	35	63	24	51	72
Remainder,						

	(7.)	(8.)	(9.)	(10.)	(11.)	(12.)
From	54 *yds.*	68 *yds.*	$27	69 *lbs.*	96 *in.*	79 *ft.*
Subtract	22 *yds.*	35 *yds.*	$16	27 *lbs.*	24 *in.*	27 *ft.*

Construct an example in subtraction having 3 orders of units, in which the numbers are abstract.

Construct two examples, in which the numbers are concrete, each having three orders of units.

Which is the minuend? Which is the subtrahend? What is the remainder in each example?

How do you prove subtraction?

46. *When any figure in the subtrahend is greater than the corresponding figure in the minuend.*

If we subtract 4 from 7 we get 3. If we add 5 to both minuend and subtrahend, and then subtract, we get the same remainder 3. Thus,

From	7
Take	4
Rem.	3

From	$7 + 5 = 12$
Take	$4 + 5 = \ \ 9$
Rem.	3

Also, add 9 to both minuend and subtrahend. Thus,

From	$7 + 9 = 16$
Take	$4 + 9 = 13$
Rem.	3

From these illustrations we see that if the same number be added to both minuend and subtrahend, the remainder is not changed.

From	5482
Subtract	2625
Rem.	2857

EXPLANATION.—Since 2 *units* are less than 5 *units*, we add 1 *ten* to 2 *units*, making 12 *units*, and subtract 5 *units* from 12 *units*, leaving 7 *units*, which we write beneath. To balance the 1 *ten* added to the minuend we add 1 *ten* to the subtrahend, making 3 *tens*, which we subtract from 8 *tens*, leaving 5 *tens*; this we write below. Since 4 *hundreds* are less than 6 *hundreds*, we add 10 *hundreds* to 4 *hundreds*, making 14 *hundreds*, from which subtract 6 *hundreds*, and write the difference, 8 *hundreds*, beneath. To balance 10 *hundreds*, equal to 1 *thousand*, added to the minuend, we add 1 *thousand* to the subtrahend, making 3 *thousands*, which we subtract from 5 *thousands* and write the remainder, 2 *thousands*, beneath, making our entire remainder 2,857.

From these illustrations and principles we deduce the following

RULE. REGRA

I. Write the less number under the greater so that units of the same order shall stand in the same column.

II. Beginning at the right hand, subtract each figure in the lower line from the one above it and set the remainder in the line below.

III. If a figure in the lower line is greater than the one above it, increase the latter by 10, perform the subtraction, and then add 1 to the next figure in the lower line.

EXAMPLES FOR WRITTEN WORK.

	(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
From	61	73	64	82	94	58
Subtract	29	48	27	76	78	39
Rem.						

	(7.)	(8.)	(9.)	(10.)	(11.)
From	578	964	887	843	765
Subtract	343	352	324	232	234
Rem.					

	(12.)	(13.)	(14.)	(15.)	(16.)
From	897 *lbs.*	415 *ft.*	679 *yds.*	$813	589 *in.*
Subtract	534 *lbs.*	203 *ft.*	234 *yds.*	$261	98 *in.*
Rem.					

Construct an example in subtraction having 3 orders of units, in which the numbers are abstract.

Construct two examples, in which the numbers are concrete, each having three orders of units.

Which is the minuend? Which is the subtrahend? What is the remainder in each example?

How do you prove subtraction?

46. ***When any figure in the subtrahend is greater than the corresponding figure in the minuend.***

If we subtract 4 from 7 we get 3. If we add 5 to both minuend and subtrahend, and then subtract, we get the same remainder 3. Thus,

From	7		From	$7 + 5 = 12$
Take	4		Take	$4 + 5 = 9$
Rem.	3		Rem.	3

Also, add 9 to both minuend and subtrahend. Thus,

From	$7 + 9 = 16$
Take	$4 + 9 = 13$
Rem.	3

From these illustrations we see that if the same number be added to both minuend and subtrahend, the remainder is not changed.

From	5482
Subtract	2625
Rem.	2857

EXPLANATION.—Since 2 *units* are less than 5 *units*, we add 1 *ten* to 2 *units*, making 12 *units*, and subtract 5 *units* from 12 *units*, leaving 7 *units*, which we write beneath. To balance the 1 *ten* added to the minuend we add 1 *ten* to the subtrahend, making 3 *tens*, which we subtract from 8 *tens*, leaving 5 *tens*; this we write below. Since 4 *hundreds* are less than 6 *hundreds*, we add 10 *hundreds* to 4 *hundreds*, making 14 *hundreds*, from which subtract 6 *hundreds*, and write the difference, 8 *hundreds*, beneath. To balance 10 *hundreds*, equal to 1 *thousand*, added to the minuend, we add 1 *thousand* to the subtrahend, making 3 *thousands*, which we subtract from 5 *thousands* and write the remainder, 2 *thousands*, beneath, making our entire remainder 2,857.

From these illustrations and principles we deduce the following

RULE.

REGRA

I. Write the less number under the greater so that units of the same order shall stand in the same column.

II. Beginning at the right hand, subtract each figure in the lower line from the one above it and set the remainder in the line below.

III. If a figure in the lower line is greater than the one above it, increase the latter by 10, perform the subtraction, and then add 1 to the next figure in the lower line.

EXAMPLES FOR WRITTEN WORK.

	(1.)	*(2.)*	*(3.)*	*(4.)*	*(5.)*	*(6.)*
From	61	73	64	82	94	58
Subtract	29	48	27	76	78	39
Rem.						

	(7.)	*(8.)*	*(9.)*	*(10.)*	*(11.)*
From	578	964	887	843	765
Subtract	343	352	324	232	234
Rem.					

	(12.)	*(13.)*	*(14.)*	*(15.)*	*(16.)*
From	897 *lbs.*	415 *ft.*	679 *yds.*	\$813	589 *in.*
Subtract	534 *lbs.*	203 *ft.*	234 *yds.*	\$261	98 *in.*
Rem.					

	(17.)	(18.)	(19.)	(20.)
From	2,843	$5,946	2,003	8,800 *ft.*
Subtract	1,678	$1,389	976	2,088 *ft.*
Rem.				

(21.)	(22.)	(23.)	(24.)	(25.)
4760	3695	87231	67087	12048
1986	1863	1009	40000	8034

26. From 287 subtract 114.

27. From 994 subtract 363.

28. From 10,841 subtract 3,009.

29. From $12,560 subtract $4,885.

30. From 115,440 *ft.* subtract 19,359 *ft.*

31. From 1,310,844 subtract 337,775.

32. From $4,478 take $989.

33. From 77,475 *yds.* take 10,994 *yds.*

34. Find the difference between $785 and $323.

35. Find the difference between 12,843 *yds.* and 2,318 *yds.*

36. Find the difference between 711,711 and 82,082.

37. What is the difference between 5,858 *ft.* and 949 *ft.*?

38. How much does 244,887 exceed 108,104?

39. $2,478 + 1,236 - (2,562 - 1,893) = ?$

TEST QUESTIONS.

What is subtraction? What is the minuend? What is the subtrahend? What is the difference, or remainder? How are the numbers written for work in subtraction? When the figures of the subtrahend are equal to or less than the corresponding figures of the minuend, how do you subtract? When a figure in the subtrahend is greater than its corresponding figure in the minuend, how do you subtract? How do you prove subtraction? Give the rule for subtraction. What is a problem? What is the sign of subtraction? How is the parenthesis used in examples of subtraction?

PRACTICAL PROBLEMS.

FOR ORAL WORK.

47. *1.* A man having $10, paid $6 for a pair of boots; how many dollars had he left?

SOLUTION.—He had the difference between $10 and $6, which is $4.

2. A boy had 20 marbles, and gave away 7 of them; how many had he left? $20 - 7 = ?$

3. Mary is 15 years old, and Jane is 9; how long before Jane will be 15? $15 - 9 = ?$

4. Two boys have together 20 *cts.*; if one has 12 *cts.*, how many has the other? $20 - 12 = ?$

5. James bought a book for 20 *cts.*, and paid a 25 *ct.* piece; how much change should he receive?

6. Charles has 50 *cts.*, he pays 10 *cts.* for a pencil and 5 *cts.* for a rubber; how many cents has he left?

FOR WRITTEN WORK.

7. A merchant bought 750 *yds.* of cloth and sold 468 *yds.* of the same; how many *yds.* had he left?

8. From a flock containing 718 sheep 432 were sold; how many remained? $718 - 432 = ?$

9. A man bought a pair of horses for $788, and sold them again for $629; how much did he lose?

10. A merchant bought a stock of goods for $1,887 and sold the same for $2,143; how much did he gain?

11. A man's income is $6,000 per annum and his expenses are $4,125; how much does he save?

FOR ORAL WORK.

48. *1.* A man 50 years old has a son 20 years old; how much older is the father than the son?

2. A man having 30 miles to travel in 2 days, goes 18 miles the first day; how far must he go the second day?

3. A woman has \$21 and spends \$12; how much money has she left? \$21 — \$12 = ?

4. John weighs 60 *lbs.* and Mary 50 *lbs.*; how much heavier is John than Mary? 60 *lbs.* — 50 *lbs.* = ?

5. A man had \$100, \$80 of which was in the bank and the rest in his pocket; how much in his pocket?

6. A man purchased a farm for \$10,000, and paid cash \$6,000, how much remained unpaid?

FOR WRITTEN WORK.

7. A merchant sells goods for \$17,480 and gains by the sale \$4,894; what did they cost him?

8. A farmer had 497 sheep, but he sold at one time 113 and at another time 98; how many had he left?

9. A merchant begins business with \$18,413; the first year he loses \$800 and the second year he gains \$976; what is he then worth?

10. A man has an income of \$2,500 per annum, and he spends \$750 for rent, \$1,200 for living expenses, and the remainder he saves; how much does he save per year?

11. A drover bought 711 sheep of one farmer, 310 sheep of another, and then sold 462; how many had he left? 711 + 310 — 462 = ?

FOR ORAL WORK.

49. *1.* A boy having 15 *cts.* spent 5 *cts.* for a pencil and 8 *cts.* for a sponge; how much money had he left?

2. A boy has \$30 in the bank; he draws out \$7 at one time and \$9 at another; how much remains in the bank?

3. A boy bought sugar for 10 *cts.* and eggs for 12 *cts.*, and gave the clerk 25 *cts.*; how much change should he receive? 25 — (10 + 12) = ?

4. A boy who had 10 marbles bought 15 more, and he then lost 12; how many had he left?

5. A man bought a horse for \$60, a harness for \$20, a wagon for \$30; he afterward sold them all for \$100; how much did he lose? (\$60 + \$20 + \$30) — \$100 = ?

FOR WRITTEN WORK.

6. From a regiment of 847 men 143 were discharged and 273 were killed in battle; how many remained?

7. A trader commences business with a capital of \$3,245; the first year he gains \$422, the second year he gains \$500; the third year he loses \$792, and the fourth year he loses \$117; how much is he then worth?

8. A man had \$13,850 in the bank, but drew out at one time \$1,872, at another time \$3,814, and at a third time \$4,811; how much had he then in bank?

9. A man gave to his four sons \$3,780; to the first he gave \$1,490, to the second he gave \$1,109, to the third he gave \$675, and to the fourth he gave the remainder; how much did he give the fourth?

10. Four men bought a tract of land, for which they paid \$8,419; the first paid \$3,815, the second paid \$2,140, the third paid \$1,480; what did the fourth pay?

FOR ORAL WORK.

50. *1.* An orchard contained 50 trees, 10 of which were peach trees, 5 pear trees, 8 plum trees, and the rest were apple trees; how many apple trees in the orchard?

2. I bought a coat for \$20, a vest for \$8, pants for \$10,

and I paid a \$50 bill; how much did I receive in return? \$50 — (\$20 + \$8 + \$10) = ?

3. A dish contained 60 peaches, Jane took 12, Susan 10, Mary 13, and John 15; how many were left in the dish?

4. Six men bought a horse for \$150; the first gave \$50, the second \$30, the third \$25, the fourth \$18, and the fifth \$10; how much did the sixth give?

5. A farmer bought a horse for \$100, and paid \$15 for keeping him; he let him enough to receive \$25 and then sold him for \$90; did he gain or lose by the bargain? How much?

FOR WRITTEN WORK.

6. A man gave to his four sons \$5,880; to the first he gave \$2,360, to the second he gave \$2,109, to the third he gave \$805, and to the fourth he gave the remainder; how much did he give the fourth? *Ans.* \$606.

7. A householder sold two houses; for the first, which cost \$3500, he received \$4760; for the second, which cost \$3735, he received \$5000; on which of the houses did he make the greater gain, and how much?

8. A person borrowed of his neighbor at one time \$355, at another time \$637, and \$403 at another time; he paid him \$977; how much did he then owe him?

9. I have a yearly income of \$10,000. I pay \$275 for office rent, \$220 for fuel, \$35 to the doctor, and \$3675 for all my other expenses; how much have I left at the end of the year?

10. A man pays \$300 for 100 sheep, \$95 for a pair of oxen, \$60 for a horse, and \$125 for a chaise; he gives 100 bushels of wheat worth \$125, a cow worth \$25, a colt

worth \$40, and pays the rest in cash; how much money does he pay?

11. If the subtrahend be 750 and the remainder 964, what is the minuend?

12. If the minuend be 60,402 and the remainder 29,475, what is the subtrahend?

13. The difference of two numbers is 607 and the greater number is 1,005; what is the less number?

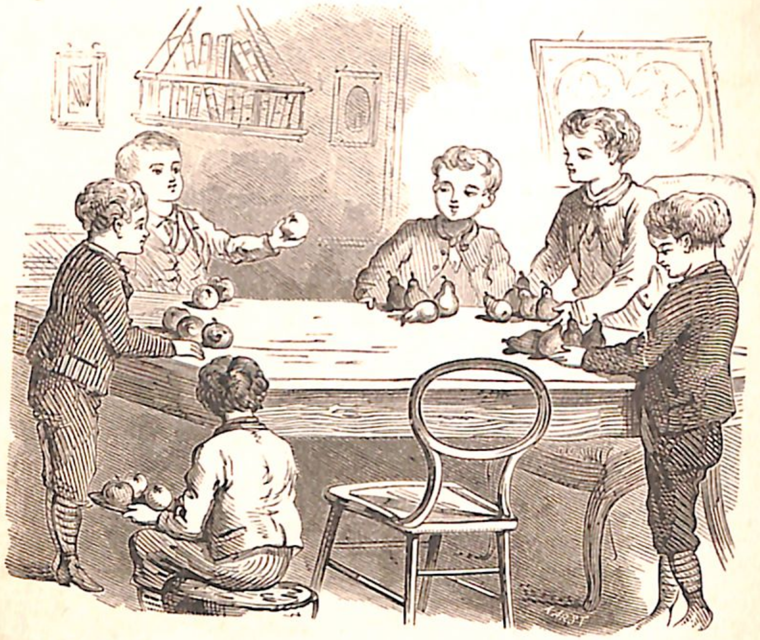
14. 7,963 + 54,923 + 27,984 — 64,937 = ?

15. 22,786 — (10,342 + 5,684) = ?

16. A man worth \$18,000 left \$4,287 to his elder son, \$3,754 to his younger son, \$3,219 to his daughter, and the remainder to his wife; what was the wife's portion?

REVIEW QUESTIONS.

What is a unit? What is a number? What is an abstract num-
ber? What is a concrete number? What is a simple number?
What is a compound number? Define notation. Define numera-
tion. Give all the methods of notation. Write seven thousand
two hundred and fifty-one by means of the Arabic notation. Write
five hundred and seventy-eight by means of the Roman notation
Numerate 7,852,643,827,462, and read the number. What is addi
tion? What numbers can be added together? Give the rule for
addition. How do you prove addition? Make the sign of addition.
Of equality. Make the sign of dollars. When dollars and cents
are written, how many orders of units are occupied by cents?
What sign is put between dollars and cents? When dollars, cents,
and mills are written, how many orders of units do mills occupy?
What is subtraction? Define minuend. Define subtrahend. Define
remainder. Make the sign of subtraction, name it, and tell how it is
used. Give the rule for subtraction. Work the following exam-
ple: 178,462—(6,895 + 18,754). When the difference and the greater
of two numbers are given, how do you find the less? Suppose the
sum of three numbers and two of them are given, how will you
find the third? Construct a problem illustrating each of the above
questions.



MULTIPLICATION.

51. *1.* In this picture there are two groups of boys and 3 boys in each group; how many boys are there in the picture?

SOLUTION.—Since there are 3 boys in one group, in 2 groups there are 2 times three boys, $3+3=6$, 3 taken twice $=6$.

2. In one group, each boy has 3 apples; how many apples have the 3 boys?

SOLUTION.—Since each boy has 3 apples, 3 boys have 3 times 3 or 9 apples, $3+3+3=9$, or 3 taken 3 times $=9$.

3. In the other group, each boy has 4 pears; how many pears have the 3 boys.

SOLUTION.—Since each boy has 4 pears, 3 boys will have 3 times four or 12 pears, $4+4+4=12$, or 4 taken 3 times $=12$.

4. How many trees in 4 rows, if there are 5 trees in each row? 4 times 5 trees are how many trees?

SOLUTION. $5+5+5+5=20$, or 5 taken 4 times $=20$.

5. How many hands have 6 boys?

SOLUTION. $2+2+2+2+2+2=12$, or 2 taken 6 times $=12$.

6. How many feet have 6 horses?

SOLUTION. $4+4+4+4+4+4=24$, or 4 taken 6 times $=24$.

7. John's father gave him 6 five-cent pieces; how many cents did the father give John?

SOLUTION. $5+5+5+5+5+5=30$, or 5 taken 6 times $=30$.

8. Bought 3 *lbs.* of sugar at 10 *cts.* a pound; how much did the sugar cost? 3 times 10 *cts.* = ?

SOLUTION. $10+10+10=30$, 10 taken 3 times $=30$.

In the first example we find the sum of 2 threes, or 3 taken twice. In the second we find the sum of 3 threes, or 3 units taken 3 times. In the fourth we find the sum of 3 fours, or 4 taken 3 times, and so on.

In the first example, how many times is 3 taken? What is the result? In the fifth example, how many times is the number 2 taken? In the sixth example, how many times is 4 taken? What number is taken 6 times in the seventh example? How many times is 10 taken in the eighth?

52. The operation of taking a number a certain number of times is called **Multiplication**.

The number to be taken is called the **Multiplicand**, and the number which shows how many times the multiplicand is taken is called the **Multiplier**.

DEFINITIONS.

53. Multiplication is the operation of taking one number as many times as there are units in the other.

54. The **Multiplicand** is the number to be taken or multiplied.

55. The **Multiplier** is the number which shows how many times the multiplicand is to be taken, or what part of it is to be taken.

56. The **Product** is the result of the multiplication.

What is multiplication? What is the multiplicand? What is the multiplier? In each of the eight examples on pages 60 and 61, tell which number is the multiplicand? Which is the multiplier? In Example 1st, 6 is the product; in Example 2d, 9 is the product; in Example 3d, 12 is the product. Tell what is the product in each of the other examples.

57. The multiplicand and multiplier are called **Factors** of the product.

58. The following is the **Sign of Multiplication**, ×. When placed between two numbers it is read *multiplied by*; thus, 3 × 2 is read *3 multiplied by 2*.

The value of the product does not depend on the order in which the factors are taken. Thus, 4 times 5 is the same as 5 times 4, as shown in the diagram; for, if we take the stars by rows, we have 4 stars taken 5 times; if we take them by columns, we have 5 stars taken 4 times; in either case there are 20 stars. The multiplier, however, must always be considered an abstract number. The multiplicand and product are like numbers, and may be abstract or concrete.

```
*  *  *  *
*  *  *  *
*  *  *  *
*  *  *  *
*  *  *  *
```


59. The **elements of multiplication** are given in the following table, called the

MULTIPLICATION TABLE.

Once	2 times	3 times	4 times
1 is 1	1 are 2	1 are 3	1 are 4
2 " 2	2 " 4	2 " 6	2 " 8
3 " 3	3 " 6	3 " 9	3 " 12
4 " 4	4 " 8	4 " 12	4 " 16
5 " 5	5 " 10	5 " 15	5 " 20
6 " 6	6 " 12	6 " 18	6 " 24
7 " 7	7 " 14	7 " 21	7 " 28
8 " 8	8 " 16	8 " 24	8 " 32
9 " 9	9 " 18	9 " 27	9 " 36
5 times	6 times	7 times	8 times
1 are 5	1 are 6	1 are 7	1 are 8
2 " 10	2 " 12	2 " 14	2 " 16
3 " 15	3 " 18	3 " 21	3 " 24
4 " 20	4 " 24	4 " 28	4 " 32
5 " 25	5 " 30	5 " 35	5 " 40
6 " 30	6 " 36	6 " 42	6 " 48
7 " 35	7 " 42	7 " 49	7 " 56
8 " 40	8 " 48	8 " 56	8 " 64
9 " 45	9 " 54	9 " 63	9 " 72
9 times	10 times	11 times	12 times
1 are 9	1 are 10	1 are 11	1 are 12
2 " 18	2 " 20	2 " 22	2 " 24
3 " 27	3 " 30	3 " 33	3 " 36
4 " 36	4 " 40	4 " 44	4 " 48
5 " 45	5 " 50	5 " 55	5 " 60
6 " 54	6 " 60	6 " 66	6 " 72
7 " 63	7 " 70	7 " 77	7 " 84
8 " 72	8 " 80	8 " 88	8 " 96
9 " 81	9 " 90	9 " 99	9 " 108

NOTE.—The Multiplication Table should be *perfectly committed to memory.*

Exercises for Mental Work.

1. $5 \times 4 = 20.$	*8.* $12 \times 6 = ?$	*15.* $9 \times 8 = ?$
2. $7 \times 8 = ?$	*9.* $7 \times 9 = ?$	*16.* $12 \times 7 = ?$
3. $6 \times 7 = ?$	*10.* $8 \times 7 = ?$	*17.* $10 \times 8 = ?$
4. $8 \times 6 = ?$	*11.* $4 \times 9 = ?$	*18.* $11 \times 9 = ?$
5. $9 \times 4 = ?$	*12.* $12 \times 4 = ?$	*19.* $12 \times 11 = ?$
6. $3 \times 7 = ?$	*13.* $9 \times 9 = ?$	*20.* $6 \times 8 = ?$
7. $6 \times 9 = ?$	*14.* $4 \times 7 = ?$	*21.* $8 \times 9 = ?$

What are the factors in the first example above? What in the fifth? What in the tenth? What in the twentieth? Name the factors of 6; of 10; of 14; and of 15.

Supply the missing factor in each of the following:

1. $7 \times \quad = 21.$	*6.* $7 \times \quad = 56.$	*11.* $12 \times \quad = 60.$
2. $\quad \times 4 = 36.$	*7.* $11 \times \quad = 121.$	*12.* $\quad \times 6 = 30.$
3. $\quad \times 9 = 54.$	*8.* $\quad \times 9 = 108.$	*13.* $\quad \times 9 = 72.$
4. $12 \times \quad = 60.$	*9.* $\quad \times 7 = 63.$	*14.* $7 \times \quad = 28.$
5. $\quad \times 6 = 42$	*10.* $12 \times \quad = 96.$	*15.* $\quad \times 8 = 32.$

If 4 marbles be taken 5 times, which is the abstract number? Which the concrete? Which is the multiplicand? Which the multiplier? What is the product? Is the product abstract or concrete?

Exercises for Oral Work.

60. *1.* If 1 *lb.* of rice costs 11 *cts.*, how much will 4 *lbs.* cost? 11 *cts.* $\times 4 =$ how many *cts.*?

2. 7 boys receive 8 *cts.* each; how many cents do all receive? 8 *cts.* $\times 7 =$ how many *cts.*?

3. What will 9 lemons cost, at 9 *cts.* each?

4. How much can a boy earn in 8 weeks, if he earn \$7 each week? $\$7 \times 8 =$ how many dollars?

5. A man walks 6 miles a day for 12 days; how far does he go? $6 \times 8 =$ how many? $6 \times 9 = ?$

6. There are 3 feet in 1 yard; how many feet in 6 yards? How many in 7 yards? In 8 yards? 10 yds.?

7. Bought 12 *yds.* of cloth at \$5 a yard; how much did it cost? How much did 10 *yds.* cost?

8. What will 4 pairs of boots cost at \$8 a pair?

9. In 1 peck there are 8 quarts; how many quarts in 9 pecks? How many in 10 pecks? In 12 pecks?

10. There are 12 inches in 1 foot; how many inches in 11 feet? In 12 *ft.*? In 9 *ft.*? In 8 *ft.*? In 7 *ft.*?

Exercises for Written Work.

1. $\$50 \times 4 = ?$	*6.* 12 *marbles* $\times 3 = ?$
2. 19 *cts.* $\times 6 = ?$	*7.* 16 *horses* $\times 4 = ?$
3. 16 *ft.* $\times 9 = ?$	*8.* 9 *sheep* $\times 2 = ?$
4. 32 *in.* $\times 3 = ?$	*9.* 8 *miles* $\times 5 = ?$
5. 8 *apples* $\times 7 = ?$	*10.* 7 *pounds* $\times 6 = ?$

Instead of writing in a horizontal line, with the sign $\times$ between the factors, it is more convenient to write the multiplier under the multiplicand.

61. ***When the multiplier consists of but one figure.***

Multiply 328 by 7.

ILLUSTRATION.

Multiplicand,	328
Multiplier,	7
Product,	2,296

EXPLANATION.—Multiplying 8 by 7, we have 56, that is, 5 *tens* and 6 *units;* we set down the 6 *units* and carry forward the 5 *tens* to the product of 2 tens multiplied by 7. Multiplying 2 *tens* by 7, we have 14 *tens*, which, increased by the 5 *tens* brought forward, gives 19 *tens*, or 1 *hundred* and 9 *tens;* we set down the 9 *tens* and carry forward the 1 *hundred*. Multiplying 3 *hundreds* by 7, we

have 21 *hundreds*, which, increased by the 1 *hundred* brought forward, gives 22 *hundreds*, or 2 *thousands* and 2 *hundreds*; this we set down. The required product is 2,296.

NOTE.—The operation of multiplying may be abbreviated, as explained in addition, by omitting the names of the figures, and simply naming the results of the successive multiplications.

RULE.

Begin at the right and multiply each figure of the multiplicand by the multiplier, setting down and carrying as in addition.

EXAMPLES.

Perform the following multiplications:

	(1.)	(2.)	(3.)	(4.)
Multiplicand,	134	318	256	808
Multiplier,	3	4	2	4
Product,	402			

	(5.)	(6.)	(7.)	(8.)
Multiplicand,	318	476	1234	4137
Multiplier,	5	7	8	9
Product,				

	(9.)	(10.)	(11.)	(12.)
Multiplicand,	417 *ft.*	1843 *in.*	$894	693 *lbs.*
Multiplier,	6	7	5	9
Product,	2502 *ft.*	*in.*	$4470	*lbs.*

(13.)	(14.)	(15.)	(16.)
3145	43214 *lbs.*	81342	$5486
8	7	6	9
25160	*lbs.*	488052	$

What is the rule when the multiplier consists of but one figure?

62. ***When the multiplier contains any number of figures.***

EXAMPLE.

1. Multiply 458 by 346.

EXPLANATION.—Here we write the numbers so that units of the same order stand in the same column. We then multiply 458 by 6, which gives 2748. We next multiply 458 by 4 *tens*, and set the first figure in the column of tens. We then multiply 458 by 3 *hundreds*, and place the first figure in the column of hundreds. Adding the partial products thus obtained, we find for the total product 158,468.

ILLUSTRATION.

Multiplicand,	458
Multiplier,	346
Partial Products.	2748 1832 1374
Product,	158468

From this illustration and explanation we deduce the following

RULE.

I. Write the multiplier under the multiplicand, so that units of the same order shall stand in the same column.

II. Begin at the right and multiply the multiplicand by each figure of the multiplier, writing the first figure of each partial product under the corresponding figure of the multiplier.

III. Find the sum of the partial products.

PROOF.—*Multiply the multiplier by the multiplicand, and if the second product equals the first the work is presumed to be right.*

EXAMPLES FOR WRITTEN WORK.

(1.)	*Proof.*	(2.)	*Proof.*
78	64	86	67
64	78	67	86
4992	4992	5762	5762

Multiply

3. 7,406 by 36.
4. 3,421 by 48.
5. 8,413 by 75.
6. 719 by 183.
7. 743 by 345.
8. 828 by 712.
9. 294 by 252.
10. 813 by 712.
11. 576784 by 64.
12. 596875 by 144.
13. 46123101 by 72.
14. 6185720 by 132.
15. 718328 by 96.
16. 679534 by 9185.
17. 86972 by 1208.
18. 1055054 by 279.

Multiply

19. 6,431 by 27.
20. 2,782 by 28.
21. 9,346 by 54.
22. 1,243 by 126.
23. 873 by 284.
24. 1,349 by 236.
25. 2,157 by 317.
26. 3,184 by 196.
27. 792 by 215.
28. 349 by 318.
29. 92 by 47.
30. 1,894 by 23.
31. 757 by 132.
32. 2,416 by 99.
33. 1,308 by 102.
34. 3,047 by 205.

What is the rule when the multiplier contains any number of figures? What is the method of proof?

63. *When one or both factors have ciphers on the right.*

Every cipher that we annex to a whole number moves each of its digits one place to the left; but this is the same as multiplying by 10.

Hence, to multiply a whole number by 10, we annex *one* cipher to the multiplicand; to multiply by 100, we annex *two* ciphers; to multiply by 1,000, we annex *three* ciphers; and so on. Thus, $75 \times 10 = 750$; $34 \times 100 = 3{,}400$; $87 \times 1000 = 87{,}000$.

If the significant figure of the multiplier followed by ciphers is greater than 1, we first multiply by the significant figure or figures, and then annex the ciphers.

Multiply 317 by 300.

ILLUSTRATION.

```
 317
 300
95100
```

EXPLANATION.—We multiply 317 by 3, which gives 951, and to this we annex two ciphers, as shown in the illustration.

EXAMPLES.

Multiply

1. 318 by 20.
2. 914 by 900.
3. 8,143 by 500.
4. 4,175 by 80.
5. 874 *yds.* by 300.
6. \$841 by 70.
7. 8,888 by 3,700.

Multiply

8. 406 by 400.
9. 516 by 800.
10. 217 by 2000.
11. 429 by 400.
12. \$27 by 601.
13. 927 by 1200.
14. 561 by 2,050.

If both factors terminate in ciphers, multiply the significant figures, and to the result annex as many ciphers as there are at the right of both factors.

Multiply 8900 by 9000.

ILLUSTRATION.

```
    8900
    9000
80100000
```

EXPLANATION.—Here we multiply 89 by 9, which gives 801, and to the result we annex five ciphers as shown in the illustration.

EXAMPLES.

15. Multiply 870 by 300.
16. Multiply 41,900 by 90.
17. 2500 by 500.
18. 5600 by 2000.

Multiply

19. 8,100 by 7,000.
20. 7,300 by 50.
21. 3,460 by 80.
22. $20,370 by 70.
23. 5,150 *yds.* by 600.

Multiply

24. 7,050 by 200.
25. 400 by 300.
26. 7100 by 50.
27. 3912 by 600.
28. 4200 *ft.* by 30.

One cipher annexed to a number moves each digit how many places to the left? This is the same as multiplying by what number? How do you multiply by 10? How by 100? How by 1,000? When the multiplier is a significant figure followed by one or more ciphers how do you multiply?

COMPOSITE NUMBERS.

64. A **composite** number is one that can be separated into other integral factors than itself and one.

Thus, 6 is a composite number, because it can be separated into the factors 2 and 3.

NOTE.—Scholars should be carefully taught to distinguish between *factors* of a number and *parts* of a number. Any number is the sum of its parts, but the product of its factors.

EXERCISES FOR ORAL WORK.

Separate into two factors each of the following numbers:

(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)	(*6.*)	(*7.*)	(*8.*)	(*9.*)	(*10.*)
10,	15,	21,	14,	22,	25,	26,	32,	40,	42.

Separate into three factors each of the following numbers:

(*11.*)	(*12.*)	(*13.*)	(*14.*)	(*15.*)	(*16.*)	(*17.*)	(*18.*)	(*19.*)	(*20.*)
8,	12,	16,	24,	27,	30,	36,	48,	56,	45.

To multiply by a composite number, we may multiply by each of its factors in succession. Thus, to multiply 118 by 24, we may multiply 118 by 6, which gives 708,

and then multiply the result by 4, which gives 2,832; this is the required product.

In the same manner multiply the following

EXAMPLES.

21. Multiply 78 by 48, (6×8).
22. Multiply 96 by 108, (9×12).
23. Multiply 413 by 56, (7×8).
24. Multiply 88 by 3×7, (21).
25. Multiply 546 by 8×12, (96).
26. Multiply 8,342 by 7×12, (84).
27. Multiply 8×5 by 7×6.
28. Multiply 15×18 by 16×12.

The product of more than two factors is called a **Continued Product**. Thus, in example 27, the number 1,680 is the continued product of 8, 5, 7, and 6.

What is a composite number? How do you multiply by the factors of a composite number? What is a continued product?

PRACTICAL PROBLEMS.

FOR ORAL WORK.

65. *1.* What will 7 hundred pounds of sugar cost at $9 a hundred? $9 × 7 = how many dollars?

2. 4 quarters make 1 yard; how many quarters in 8 yards? 4 times 8 equals how many?

3. If a man earn $7 a week, how much will he earn in 10 weeks? How much in 12 weeks?

4. How many yards of cloth in 7 pieces, each piece containing 10 yards? How many in 8 pieces?

5. What will 5 barrels of flour cost at $8 a barrel?

FOR WRITTEN WORK.

6. What will 38 barrels of flour cost at $13 a barrel?
Ans. $494.

7. What is the cost of 675 *lbs.* of cheese at 14 cents a pound? At 12 cents a pound? At 11 cents a pound?

8. A farmer sold 211 bushels of potatoes at 74 cents a bushel; how much did he receive?

9. What is the cost of 786 quarts of milk at 9 cents a quart? What at 10 cents a quart?

10. If $1 will buy 21 tickets, how many will $37 buy?

FOR ORAL WORK.

66. *1.* In one yard there are 3 feet; how many feet in 12 yards? In 9 yards? In 7 yards? In 11 yards?

2. How many feet in 6 yards and 2 feet? In 7 yards and 2 feet? In 6 yards and 1 foot?

3. If one quarter of a yard of beaver cloth costs $2, what will 1 yard cost? What will 2 yards cost?

4. If 4 bushels of wheat make one barrel of flour, how many bushels will be required to make 9 barrels?

5. A gentleman bought 10 yards of silk at $2 a yard, and 6 pairs of stockings at 50 *cts.* a pair; how much should he pay for the goods?

FOR WRITTEN WORK.

6. If $1 will buy 4 *lbs.* of butter, how many pounds will $82 buy? How many *lbs.* will $37 buy?

7. If $1 will buy 8 *lbs.* of sugar, how many pounds will $17 buy? How many will $13 buy?

8. There are 3,600 seconds in 1 hour; how many seconds are there in 24 hours, or 1 day?

9. How many seconds are there in two days?

10. If a chest of tea contains 64 *lbs.* and each pound is worth 70 cents, what will be the value of 18 chests?

11. A drover bought 74 head of cattle at $82 a head, and sold the lot for $7,500; how much did he make?

FOR ORAL WORK.

67. *1.* 10 decimeters make 1 meter; how many decimeters make 6 meters?

2. 10 centimeters make one decimeter; how many centimeters in 8 decimeters?

3. 10 millimeters make one centimeter; how many millimeters make 9 centimeters?

4. On a chess-board there are eight rows of squares and eight squares in each row; how many squares are there on the board?

5. Two men start from the same place and travel in opposite directions; one travels 2 miles an hour, the other travels 3 miles an hour; how far apart will they be at the end of 5 hours?

6. Two men start from the same place and travel the same way; one travels 2 miles an hour and the other 3 miles an hour; how far apart will they be at the end of 8 hours?

7. Bought 3 meters of linen at $2 a meter, 7 meters of silk at $3 a meter, 5 meters of ribbon for $4, some crape for $2, and gave the merchant 4 ten-dollar bills; how much change should I receive back?

FOR WRITTEN WORK.

8. A farmer has 3 flocks of sheep, numbering respectively 50, 60, and 75 head, and each sheep yields 4 *lbs.* of

wool; what is the value of his wool crop when wool is worth 36 cents a pound?

9. Two couriers travel toward each other, the first at the rate of 35 miles and the second at the rate of 42 miles a day; at the end of 9 days they are separated by 411 miles; how far apart were they at first?

10. A person bought 30 *yds.* of muslin at 20 *cts.* a yard, 4 *yds.* of silk at $1.75 a yard, and 14 books at 77 cents each; what was the amount of his bill?

11. What is the difference between 118 times 327 and 211 times 82?

12. $(2134 + 506) \times (1800 - 500) = ?$

13. $(32 \times 6) + (48 \times 9) - (17 \times 4 - 3) + 160 = ?$

14. $(\$2478 - \$1032) \times (2041 + 453) \times 9 - 7 = ?$

15. What is the sum of 512 times 384, and 81 times 611?

16. $(3042\ yds. - 2106\ yds. + 218\ yds.) \times (354 - 214) \times (27 + 3) = ?$

17. $2304 + 38 + (640 - 84) \times 16 - 6 = ?$

Ans. 11,232.

REVIEW QUESTIONS.

Recite the multiplication table. What is multiplication? What is the multiplicand? What is the multiplier? What is the product? Make the sign of multiplication; tell how it is used and how it is read. Which of the two factors is always considered abstract? In the operation of multiplication how are the multiplier and multiplicand written when the sign is not used? Give the rule when the multiplier consists of but one figure. Give the rule when the multiplier consists of more than one figure. What is meant by the factors of the product? What is a composite number? How do you multiply by the factors of a composite number? What is the process when ciphers occur on the right of one or both factors. What is the use of the parentheses in Examples 12, 13, 14 and 16.

DIVISION.

68. How many boys in this picture?

Into how many groups are they divided? How many boys in each group?

1. If 10 boys are divided into two equal groups, how many boys are there in each group?

2. If 15 apples are separated into 3 equal piles, how many apples in each pile?

3. If 12 pears are divided among (Entre) four boys, how many pears will each boy receive?

4. If 20 cents will buy 5 oranges, how many cents will buy 1 orange?

5. If a man earns (Gana) \$18 in 6 days, how many dollars does he earn in 1 day?

6. In 2 days there are 48 hours, how many hours in 1 day?

7. If 3 yards of silk cost \$12, what will 1 yard cost?

8. There are 24 boys in 2 classes, with an equal number in each, how many boys in each class?

9. Paid 30 cents for 5 oranges, how many cents did 1 orange cost?

10. How many barrels of apples can be bought (Comprado) for \$40, if each barrel costs \$5.

In the first example we are required to find one of two equal parts of ten. In the third we find the number of equal parts in 12, each of which contains 3 units.

Hence, in division, we aim at **one of two objects**; either to find the *number of units in each of the equal parts* of a given number, or the *number of equal parts* into which a given number is to be divided.

The number divided is called the **Dividend.** The number which shows into how many parts the dividend is divided is called the **Divisor.** That which shows how many times the divisor is contained in the dividend is called the **Quotient.**

DEFINITIONS.

69. Division is the operation of finding how many times one number is contained in another, or of finding one of the equal parts of a number.

70. The **Dividend** is the number to be divided.

71. The **Divisor** is the number by which the dividend is to be divided.

72. The **Quotient** is the result of the division, and shows how many times the divisor is contained in the dividend.

Examine carefully the 10 examples given, pages 75 and 76, and tell which is the *dividend* in each example, which the *divisor*, and what is the *quotient*.

SIGNS OF DIVISION.

73. There are *three methods* of indicating division.

1. By a *horizontal line* with a *point or period* above and below it; thus, ÷. This sign, when standing between two numbers, shows that the first is to be divided by the second; thus, $8 \div 2$ is read 8 divided by 2.

2. By a *horizontal line* with the *dividend written above*, and the *divisor below*; thus, $\frac{8}{2}$, read 8 divided by 2.

3. By a *curved line* with the *divisor at the left*, and the *dividend at the right*; thus, 2)8, read 8 divided by 2.

Write the expression 16 divided by 2, by each of the three methods.

Read the following examples:

$12 \div 3 = 4.$	$48 \div 6 = 8.$	$\frac{72}{9} = 8.$	2)92 = 46.
$24 \div 2 = 12.$	$84 \div 7 = 12.$	$\frac{63}{7} = 9.$	3)36 = 12.
$35 \div 7 = 5.$	$100 \div 10 = 10.$	$\frac{54}{9} = 6.$	5)45 = 9.

Elements of division in which the divisors are graded from 1 to 12 are given in the following

DIVISION TABLE.

1 in			2 in			3 in			4 in		
2	2	times	4	2	times	6	2	times	8	2	times
3	3	"	6	3	"	9	3	"	12	3	"
4	4	"	8	4	"	12	4	"	16	4	"
5	5	"	10	5	"	15	5	"	20	5	"
6	6	"	12	6	"	18	6	"	24	6	"
7	7	"	14	7	"	21	7	"	28	7	"
8	8	"	16	8	"	24	8	"	32	8	"
9	9	"	18	9	"	27	9	"	36	9	"
5 in			6 in			7 in			8 in		
10	2	times	12	2	times	14	2	times	16	2	times
15	3	"	18	3	"	21	3	"	24	3	"
20	4	"	24	4	"	28	4	"	32	4	"
25	5	"	30	5	"	35	5	"	40	5	"
30	6	"	36	6	"	42	6	"	48	6	"
35	7	"	42	7	"	49	7	"	56	7	"
40	8	"	48	8	"	56	8	"	64	8	"
45	9	"	54	9	"	63	9	"	72	9	"
9 in			10 in			11 in			12 in		
18	2	times	20	2	times	22	2	times	24	2	times
27	3	"	30	3	"	33	3	"	36	3	"
36	4	"	40	4	"	44	4	"	48	4	"
45	5	"	50	5	"	55	5	"	60	5	"
54	6	"	60	6	"	66	6	"	72	6	"
63	7	"	70	7	"	77	7	"	84	7	"
72	8	"	80	8	"	88	8	"	96	8	"
81	9	"	90	9	"	99	9	"	108	9	"

74. One object of division is to divide a given number into equal parts.

A————————————B

Let AB be a line one foot long; if we divide it into two equal parts, each part is one-half of a foot.

Division may be expressed by writing the dividend above a horizontal line, and the divisor below; hence, 1 divided by 2 may be written $\frac{1}{2}$.

As a quotient, $\frac{1}{2}$ is read *one-half*.

If the same line is divided into 3 equal parts, we have $\frac{1}{3}$ (1 divided by 3), which as a quotient is read *one-third*.

If we divide it into 4 equal parts, we have $\frac{1}{4}$ (1 divided by 4), read as a quotient *one-fourth*.

NOTE.—The quotient of 2 by 3 may be written $\frac{2}{3}$; this is *one-third* of 2, or it is *two-thirds* of 1. The quotient of 3 by 7 may be written $\frac{3}{7}$; this is *one-seventh* of 3, or it is *three-sevenths* of 1. The expression $\frac{2}{3}$ is read *two-thirds*; $\frac{3}{7}$ is read *three-sevenths*; $\frac{4}{9}$ is read *four-ninths*; $\frac{11}{13}$ is read *eleven-thirteenths*; and so on. Expressions of the kind just explained are called *fractions*.

75. A Fraction is one or more equal parts of a unit.

Read the following fractions:

$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{2}{7}$, $\frac{3}{8}$, $\frac{4}{9}$, $\frac{1}{10}$, $\frac{2}{5}$,
$\frac{2}{11}$, $\frac{3}{13}$, $\frac{7}{12}$, $\frac{5}{16}$, $\frac{7}{18}$, $\frac{9}{19}$, $\frac{3}{20}$, $\frac{8}{21}$, $\frac{7}{8}$, $\frac{9}{40}$.

Write by means of figures:

One-half, One-third, One-fourth, Two-fifths, One-sixth, Three-sevenths, One-eighth, Two-elevenths, Five-ninths, Seven-fifteenths, Nine-elevenths, Two-thirds, Seventeen-twentieths, Eight-seventeenths, Five-ninths, Nineteen-thirtieths, Two-thirteenths, Twenty-fifteenths, and Ninety one-hundredths.

Supply the missing numbers in the following exercises:

42 ÷ = 6.	81 ÷ = 9.	72 ÷ = 9.
÷ 8 = 4.	44 ÷ = 4.	84 ÷ = 7.
56 ÷ 8 =	÷ 6 = 9.	÷ 8 = 12.
63 ÷ = 3.	÷ 4 = 12.	÷ 11 = 12.
÷ 5 = 7.	60 ÷ = 12.	÷ 7 = 9.
72 ÷ 12 =	84 ÷ = 14.	100 ÷ 10 =

The quotient of any number by 2 is one-half of that number. The quotient of a number by 3 is one-third of the number; by 4 is one-fourth of the number; by 5 is one-fifth; by 6 is one-sixth of the number, etc.

What is one-fourth of 12? What is one-sixth of 18? One-fifth of 20? One-seventh of 14? One-ninth of 27? One-third of 21? One-eighth of 40?

What is division? What is the dividend? What the divisor? What the quotient? Make the three signs of division, and illustrate each? What is the quotient of 1 divided by 2? Of 2 divided by 3? What is a fraction? How do we express the quotient of a less number divided by a greater? If 1 be divided into 2 equal parts, what is each part called? If into 3, what is each part called? What, if into 6 equal parts?

Examples for Oral Work.

76. *1.* How many apples, at one cent each, can you buy for 5 cents?

Solution.—As many apples as there are 1's in 5, or 5 apples.

2. How many marbles, at 2 cents each, can you buy for 4 cents?

Solution.—As many marbles as there are 2's in 4, or 2 marbles.

3. How many pears, at 3 cents each, can you buy for 6 cents? How many for 18 *cts.*?

4. How many peaches, at 4 cents each, can be bought for 12 cents? How many for 24 *cts.*?

5. If I divide 15 apples among 5 boys, giving each an equal number, how many apples will each boy receive?

6. If a man travel 6 miles an hour, how many hours will it take him to travel 18 miles? 18 ÷ 6 = ?

7. In an apple orchard there are 21 trees, and 7 trees in each row; how many rows in the orchard? 21 ÷ 7 = ?

8. A man paid $24 for 8 boxes of oranges; how much did he pay for each box? 24 ÷ 8 = ?

9. How many pairs of boots, at $9 a pair, can be bought for $36? How many for $45? $\frac{36}{9} = ?$ $\frac{45}{9} = ?$

10. If I divide a line 1 foot long into 2 equal parts, how long is each part?

11. If I divide 1 apple into 3 equal pieces, what part of the apple is each piece?

12. What is the quotient of 2 divided by 3? Of 3 divided by 4? Of 5 divided by 6? Of 7 divided by 8?

SHORT DIVISION AND LONG DIVISION.

77. There are two methods of performing the operations of division: ***1.*** *Short Division,* in which the divisor does not exceed 12; and ***2.*** *Long Division,* in which the divisor exceeds 12.

In **Short Division** much of the work is carried on mentally; in **Long Division**, the different steps of the operation are written out.

SHORT DIVISION.

78. Let it be required to divide 19,224 by 4.

ILLUSTRATION.

		Dividend,
Divisor,	4) 19224	
Quotient,	4806	

EXPLANATION.—Because 1 is less than 4, we divide 19 by 4; this gives a quotient 4, and a remainder 3; we set 4 under the 9, and to 3 we annex the following figure of the dividend, which gives 32. The quotient of 32 divided by 4 is 8; this we set under the 2. Since 2, the next figure of the dividend, is less than 4, we put a cipher in the quotient, and to 2 we annex the following figure of the dividend, which gives 24. Dividing 24 by 4, we find a quotient 6, which we write under 4. Hence, the required quotient is 4806.

EXAMPLES IN SHORT DIVISION.

(*1.*)	(*2.*)	(*3.*)	(*4.*)
5) 785	6) 618	7) 1561	8) 2736

79. ***When there is a Fraction in the quotient.***

Let it be required to divide 459 by 4.

ILLUSTRATION.

4) 459
$114\frac{3}{4}$

EXPLANATION.—Since 4 is contained in 4 once, we write 1 uuder 4 for the left-hand figure of the quotient. We multiply the divisor 4 by 1, and subtract the product mentally from 4 in the dividend, and have no remainder. We then divide 5 by 4 and obtain 1 for a quotient, which we multiply by the divisor and subtract the product mentally from 5; this leaves 1 for a remainder. To this remainder we annex 9, the next figure of the dividend, making 19. We divide 19 by 4, obtaining 4 for a quotient, which multiplied by the divisor, gives 16; this we subtract mentally from 19 and obtain 3 for a remainder. The whole dividend has now been divided by 4, except 3. We have learned that a less number can be divided by a greater by writing the divisor under the dividend with a line between; hence 3 divided by 4 is $\frac{3}{4}$, which we place at the right of 4 in the quotient, giving $114\frac{3}{4}$; read one hundred fourteen and three fourths.

In this manner work the following

EXAMPLES.

(*5.*)	(*6.*)	(*7.*)	(*8.*)	(*9.*)
7) 856	5) 324	3) 4762	9) 992	6) 3214
$122\frac{2}{7}$				

RULE FOR SHORT DIVISION.

I. Write the divisor on the left of the dividend, and separate them by a line.

II. Divide the first figure of the dividend by the divisor, and write the quotient below; or if the first figure is less than the divisor, divide the first two figures, and write the quotient under the second.

III. If there is a remainder after any division, annex to it the next figure of the dividend, and divide as before.

IV. If any partial dividend is less than the divisor, write 0 *for the quotient figure, and annex the next figure of the dividend, for a new dividend.*

V. If there is a remainder, after dividing the last figure, write the divisor under it, and annex the result to the quotient.

PROOF.—*Multiply the quotient by the divisor, and if the result is equal to the dividend, the work is correct.*

EXAMPLES.

10. Divide 5,408 by 2.
11. Divide 9,147 by 3.
12. Divide 16,146 by 5.
13. Divide 5,124 by 4.
14. Divide 63,241 by 5.
15. Divide 1,981 by 7.
16. Divide 3,475 by 9.
17. Divide 113,214 by 11.

LONG DIVISION.

80. Let it be required to divide 2,756 by 26.

ILLUSTRATION.

```
26 ) 2756 ( 106
     26
     ---
      156
      156
      ---
```

EXPLANATION.—We first say, 26 in 27, once, and place 1 in the quotient. Multiplying the divisor by one, subtracting, and bringing down the 5, we have 15 for the first partial dividend. We then say, 26 in 15, 0 times, and place the 0 in the quotient. We then bring down the 6, and find that the divisor is contained in 156, 6 times.

If the dividend contains dollars and cents, point off two figures on the right of the quotient for cents. If it contains dollars, cents, and mills, point off three figures on the right of the quotient.

RULE FOR LONG DIVISION.

I. Find how many times the divisor is contained in the fewest possible figures on the left of the dividend, for the first figure of the quotient; multiply the divisor by this figure, and subtract the product from the figures used.

II. To the remainder annex the following figure of the dividend, and divide the result by the divisor, for the second figure of the quotient; or, if the result is less than the divisor, put a cipher in the quotient, annex another figure, and proceed as before.

III. Continue the operation till all the figures of the quotient have been found.

IV. If there is a remainder after the last figure is brought down, write the divisor under it and annex the result to the quotient.

The method of proof is the same as for short division.

In applying the preceding rule, it is convenient to write the divisor on the left and the quotient on the right of the dividend. Should there be a remainder after the last figure of the quotient is found, it is to be treated as explained in short division.

EXAMPLES.

Divide	Divide	Divide
1. 854 by 25.	*9.* 17,808 by 48.	*17.* 3,894 *lbs.* by 33.
2. 11,232 by 36.	*10.* 8,856 by 82.	*18.* 3,476 *yds.* by 44.
3. 836 by 22.	*11.* 20,962 by 94.	*19.* 35,638 by 103.
4. 1,674 by 31.	*12.* 16,340 by 76.	*20.* 29,890 by 122.
5. 2,944 by 46.	*13.* $870 by 15.	*21.* 13,610 by 214.
6. 5,184 by 27.	*14.* $9,504 by $16.	*22.* 2,636 by 47.
7. 19,032 by 61.	*15.* $12,972 by 23.	*23.* 3,009 by 32.
8. 22,274 by 55.	*16.* 6,475 *lbs.* by 25.	*24.* 3,060 by 235.

NOTE.—If the entire dividend is divided there is no *final remainder*, hence we do not speak of a remainder in connection with the answer. Thus, if 13 is divided by 2 the quotient is $6\frac{1}{2}$, not "6 and a remainder 1."

In the latter case only 12 is divided by 2, the remainder 1 is not divided.

81. ***When there are ciphers on the right of the divisor.***

Divide 354,216 by 100.

FIRST SOLUTION,

```
100)354216(3542 16/100
    300
    ---
     542
     500
     ---
      421
      400
      ---
       216
       200
       ---
        16
```

EXPLANATION.—By this solution it will be seen that if we had removed as many figures from the right of the dividend as there are ciphers on the right of the divisor, the remaining figures of the dividend would be the same as the integral figures of the quotient, and the figures removed are the last remainder. This remainder is divided by writing the divisor under it as directed in the second method of expressing division.

If the divisor alone terminates in ciphers, we point them off, and also point off the same number of figures from the right of the dividend; we then divide the remaining part of the dividend by the significant part of the divisor, and annex to the last remainder the figures pointed off from the dividend. This remainder, with the entire divisor written beneath, is annexed to the quotient, and becomes a part of it.

SECOND SOLUTION.

$$\begin{array}{l} 25,00)378,43(15\frac{343}{2500} \\ \quad\;\; 25 \\ \quad\;\; \overline{128} \\ \quad\;\; 125 \\ \textit{Rem.}\ \overline{343} \end{array}$$

EXPLANATION.—We point off the two ciphers in the divisor, and also two figures from the right of dividend; we then divide 378 by 25, which gives a quotient 15, and a remainder 3; to this we annex the figures cut off from the dividend, which gives 343; but, 343 has not been divided; to divide it, we write the entire divisor under it. Hence, the final result is $15\frac{343}{2500}$.

EXAMPLES.

1. Divide 8,734 by 400.
2. Divide 34,121 by 6,000.
3. Divide 184,381 by 900.
4. Divide 37,564 by 2,500.
5. Divide 272,543 by 16,000.
6. Divide 36,452 by 1,500.

If both dividend and divisor terminate in ciphers, we strike out from each, as many as are common to both. Thus, the quotient of 16,000 by 400 is the same as the quotient of 160 by 4. Striking out two ciphers is the same as dividing both dividend and divisor by 100.

When dollars, cents, and mills are divided by 10, 100, 1000, etc., the point is moved to the left as many places as there are ciphers in the divisor. If there is not a sufficient number of figures at the left of the point, supply the deficiency by prefixing ciphers.

7. Divide 1,500 by 300.
8. Divide 21,000 by 700.
9. Divide 815,000 by 5,000.
10. Divide 62,500 by 250.
11. Divide 5,120 by 1,600.
12. Divide 67,470 by 3,000.

TEST QUESTIONS.

The quotient of any number by 2, is what part of that number? The quotient of any number by 3, is what part of that number? By 4? By 6? By 7? By 8? By 15? By 20? In methods of work there are how many cases in division? What are they? How are the operations carried on in short division? How in long division? Illustrate short division by an example of your own. Give the rule for short division. If the dividend is composed of dollars and cents, how many figures do you point off on the right of the quotient? If composed of dollars, cents, and mills, how many do you point off? Write an example in long division, perform the work, and explain each step in the process. Give the rule for long division. How do you prove division? How do you perform the work when there are ciphers on the right of the divisor? How, when there are ciphers on the right of both divisor and dividend?

PRACTICAL PROBLEMS.

82. In solving problems in division, we proceed as though both dividend and divisor were abstract numbers, and then determine the unit of the answer from the nature of the problem.

FOR ORAL WORK.

1. At \$6 a yard, how many yards can be bought for \$54?

SOLUTION.—As many as 6 is contained in 54, which is 9; hence, 9 yards can be bought for \$54.

2. How many dozen eggs can be bought for 96 cents, at 12 cents a dozen? How many for 108 *cts.*?

3. If a man can dig a ditch 56 yards long in 7 days, digging an equal number each day, how many yards does he dig in a day?

4. At $6 a ton for coal, how many tons can be bought for $72? How many for $36? For $48?

5. If 7 yards of ribbon cost 70 cents, what will 1 yard cost? What will 2 yards cost? 5 yards?

FOR WRITTEN WORK.

6. How long will it take a man to walk 1,404 miles at the rate of 27 miles a day? How long, at the rate of 81 miles a day? At the rate of 108 miles a day?

7. A man earns $1,924 in 52 weeks; how much does he earn a week? How much if he earns $3,848 in 52 weeks? How much if he earns $5,772 in 52 weeks?

8. How long will it take a steamer to sail 2,880 miles, if she sails 240 miles a day? How long, if she sails 120 miles a day? How long if she sails 60 miles a day?

9. There are 60 seconds in one minute; how many minutes in 8,640 seconds? How many in 4,320 seconds?

10. If 75 horses cost $21,225, what is the cost of each horse? What, if 150 horses cost $21,225?

FOR ORAL WORK.

83. *1.* If 9 *lbs.* of sugar cost $1.08, what will 1 *lb.* cost?

2. Paid $36 for 12 sheep; what did each sheep cost?

3. There are 4 gills in 1 pint; how many pints in 48 gills? In 96 gills? In 192 gills?

4. There are 8 quarts in 1 peck, how many pecks in 72 quarts? How many in 96 quarts? In 192 quarts?

5. If you have 7 pecks of nuts, and put them in three-quart bags, filling each bag, and then you give me what are over; how many bags will you need, and how many quarts will you give me.

FOR WRITTEN WORK.

6. If two persons start from the same point and travel in opposite directions, one at the rate of 17 miles a day, and the other at the rate of 25 miles a day, how long will it be before they are 504 miles apart?

7. If 3 horses cost $720; what will one horse cost, and what will 13 horses cost, at the same rate?

8. There are 24 hours in 1 day and 7 days in 1 week; how many weeks are there in 39,984 hours?

9. A farmer sold 4 pairs of oxen for $214 a pair, 13 cows for $43 each, and 2 horses for $118 each; after paying a debt of $251, he bought with the remainder 20 acres of land; what did the land cost per acre?

10. A clerk received $7,500 salary for 5 years services; he spent $3,900 for board, $1,210 for clothing, and deposited the rest in the bank; how much did he spend each year for board? how much each year for clothing? how much did he deposit each year in the bank?

FOR ORAL WORK.

84. *1.* If 3 quarts of berries cost 36 cts., what do 5 quarts cost?

SOLUTION.—If 3 quarts of berries cost 36 cts., 1 quart costs $\frac{1}{3}$ of 36 cts., or 12 cts., and 5 quarts cost 5 times 12 cts., or 60 cts.

2. If 4 yards of cloth cost $12, what do 9 yards cost?

3. Bought 9 barrels of flour at $6 a barrel, and paid for it in coal at $3 a ton; how many tons of coal did it take? $(9 \times 6) \div 3 = ?$

4. In one gallon there are 4 quarts. If I buy a quart of molasses at 48 cts. a gallon, and pay a 25-cent piece, how much change should I receive? $25 - (48 \div 4)$.

5. How much will one-half a gallon of vinegar cost at 24 *cts.* a gallon? At 32 *cts.* a gallon?

6. How much does $\frac{1}{4}$ of an acre of land cost at $36 an acre? At $96 an acre? At $192?

7. 10 meters make one decameter; how many decameters in 84 meters? In 49 meters?

8. If you had $67, how much flour could you buy at $5 a barrel? How much at $7 a barrel?

9. Four pecks make 1 bushel. If you buy a bushel of apples for 84 cts, what is the cost of half a peck?

10. 5 men bought a horse for $75, paying equal shares; if they sell the horse for $40, how much will each man lose? $(\$75 - \$40) \div 5 = ?$

FOR WRITTEN WORK.

11. A grocer sold 64 *lbs.* of sugar at 14 cents a *lb.*, and 4 *lbs.* of tea at 96 cents a *lb.*, for which he was paid in butter at 32 cents a *lb.*; how many pounds of butter did he receive?

12. What number multiplied by 3 will give the same product as 27 multiplied by 7?

13. A grocer buys 7,381 *lbs.* of cheese, at 8 cents a pound, and pays for it in coffee at 22 cents a pound; how much coffee does he give for the cheese?

14. If flour costs $14 a barrel, how much can be bought for $1,358?

15. A merchant sold 4 pieces of cloth: The first two pieces contained 45 *yds.* each, the third contained 47 *yds.*, and the fourth contained 53 *yds.*; for the whole he received $760; how much did he receive per yard?

16. There are $750 in 4 bags; the first contains $115, the second contains $236; the third contains $60 less than the first and second together; how much does the fourth contain?

17. A grocer packs 789 *lbs.* of butter, which fills 17 tubs and 7 *lbs.* over; how much does he put in each tub?

18. A farmer sold a farm for $18,050; he sold 50 acres for 60 dollars an acre, and the remainder at 50 dollars an acre; how much land did he sell?

19. A merchant bought a hogshead of molasses, containing 96 gallons, at 35 cents per gallon; but 26 gallons leaked out, and he sold the remainder at 50 cents per gallon; did he gain or lose, and how much?

20. Mr. Bailey has 7 calves, worth 4 dollars apiece, 9 sheep, worth 3 dollars apiece, and a fine horse, worth 375 dollars. He exchanges them for a yoke of oxen, worth 125 dollars, and a colt, worth 65 dollars, and takes the balance in hogs, at 8 dollars apiece; how many hogs does it take?

21. The distance from Chicago to San Francisco is 2,448 miles; how long will it take a man to walk the whole distance at the rate of 24 miles a day?

22. A man bought a farm for $3,612; he sold half of it at $56 an acre, and received $2,408 for the half he sold; how many acres did he buy, and what did he give per acre?

RECAPITULATION AND GENERAL PRINCIPLES.

NOTATION.

85. A **Unit** is one, or a single thing.

A **Number** is a unit, or a collection of units.

The **Simple Value** of a figure is the value it expresses when standing alone, or in the units place.

The **Local Value** is that which it has when standing in any particular place. Thus, the value taken of 2 in the first place is 2 *units,* in the second place it is 2 *tens,* in the third place it is 2 *hundreds,* and so on.

Every place in a number not occupied by a significant figure must be filled by a *cipher.*

A **Rule** is a brief direction for performing work.

A **Scale** is an order of progression on which any system of notation is founded.

A **Uniform Scale** is one in which the law of progression is the same throughout, as in the Arabic notation.

A **Varying Scale** is one in which the law of progression is changed at every step, as in the notation of English money.

ADDITION.

86. Only similar numbers can be added together.

SUBTRACTION.

87. The **Minuend** and **Subtrahend** must have the same unit, or they must be capable of being reduced to the same unit.

The same number added to or subtracted from both minuend and subtrahend, does not change the value of the remainder.

GENERAL PRINCIPLES.

MULTIPLICATION.

88. The **Multiplier** is always as an abstract number.

The **Multiplicand** and **Products** are like numbers.

The multiplier and multiplicand are together called **Factors** of the product.

Multiplying either factor by any number, *multiplies the product* by the same number.

The product of a number multiplied by itself is called the **Square of the number.**

Multiplying both factors by the same number is equivalent to multiplying the product by the square of that number.

Multiplication may be proved by dividing the product by either factor; if the quotient is equal to the other factor, the work is supposed to be right.

DIVISION.

89. Multiplying the dividend by a number, *multiplies the quotient* by that number.

Multiplying the divisor by a number, *divides the quotient* by that number.

Multiplying both dividend and divisor by the same number does not change the quotient.

Dividing the dividend by a number, *divides the quotient* by that number.

Dividing the divisor by a number, *multiplies the quotient* by that number.

Dividing both divisor and dividend by a number, does not change the value of the quotient.

When the quotient of one number divided by another is integral, the dividend is said to be *exactly* divisible by the divisor, and the divisor is called an **Exact Divisor** of the dividend.

REVIEW QUESTIONS ON THE FUNDAMENTAL RULES, PRINCIPLES, Etc.

Define arithmetic. Write a *unit*. Write a *number* greater than 1. Give an example of an abstract number. Give an example of a concrete number. Write a number containing six orders of units. Separate 123468975321 into periods, and name each period. What numbers can be added together? Give the rule for addition. What is an arithmetical scale? Give an example of a uniform scale. Name the given numbers in subtraction. Define minuend. Define remainder. Construct an example in subtraction. Work it and prove it, explaining each step in the process. From 9472, subtract 2645, and explain each step. What is multiplication? Work the following problem, explain the operation, and tell how many, and what fundamental rules are used in the solution: Two persons start from the same place, and travel in the same direction; one travels at the rate of 6 miles an hour, the other at the rate of 9 miles an hour: if they travel 8 hours a day, how far will they be apart at the end of 17 days? How far, if they travel in opposite directions? Prove the work. Define product; define factors; define multiplicand; define multiplier; define divisor; define dividend; define quotient. Make all the signs of division. Give the rule for long division. Wherein does short division differ from long division? If the divisor and dividend are concrete numbers, will the quotient be concrete or abstract? Divide 1,041,835 by 204, and explain each step in the operation. If the dividend is multiplied by any number, the divisor remaining unchanged, how is the quotient effected? If the divisor is multiplied, and the dividend remains unchanged, how is the quotient effected?

PROPERTIES OF NUMBERS.

DEFINITIONS.

90. Properties of a number are qualities that necessarily belong to it.

Exercises for Oral Work.

1. If 24 is the sum of two numbers, and one of them is 13, what is the other?

2. What two factors besides the number itself and 1, will produce the product 14? What will produce 15? 21?

3. What three numbers used as factors will produce the product 30? What three will produce 45? What 36?

DEFINITION.

91. An **Exact Divisor** of a number is one that will give an integral number only for a quotient; 2 is an exact divisor of 4, of 6, of 8, of 10, etc.

4. What is the smallest number except 1, that will exactly divide all of the numbers, 10, 12, 14 and 16?

5. What is the smallest number except 1, that will exactly divide 15, 12, 18, and 21?

6. What is the largest number that will exactly divide the numbers 12, 18, and 24?

7. What is the largest number that will exactly divide 18, 27, and 36?

8. Name an exact divisor of 6 and 12; of 9 and 18; of 24 and 36; of 24 and 32; of 28 and 36.

9. Name the largest exact divisor of 6 and 12; of 9 and 18; of 20 and 30; 24 and 32; 28 and 36.

DEFINITION.

92. **A Prime Number** is one that has no exact divisor except itself and one; as 2, 3, 5, 7.

1. Name all the prime numbers between 1 and 10; between 10 and 20; 20 and 30; 30 and 40; 40 and 50; 50 and 60; 60 and 70; 70 and 80; 80 and 90; 90 and 100.

2. Write all the prime numbers between 1 and 100.

DEFINITIONS.

PAR

An **Even Number** is a number exactly divisible by 2.

Impare

93. An **Odd Number** is one that is not exactly divisible by 2.

What is a composite number? Name the composite numbers between 1 and 20; name the prime factors of each.

The process of separating composite numbers into factors is called **Factoring.**

Exercises for Oral Work.

1. Find the prime factors of 4; of 6; of 8; of 12; of 15; of 20.

2. Find the prime factors of the *sum* of 4 and 6; 5 and 7; 8 and 7; 9 and 3; 10 and 5.

3. Find the prime factors of the product of 4 and 6; 3 and 9; 2 and 8; 4 and 5; 6 and 7.

4. What are the prime factors of 12? 10? 16? 20? 26? 28? 30? 32? 35? 36?

5. What are the prime factors of 21? 28? 32? 36? 38? 42? 45? 49? 50?

6. What are the prime factors of 27? 25? 35? 49? 44? 52? 60? 56?

Any composite number may be resolved into prime factors by the following

RULE.

Divide the number by one of its prime factors; then divide the quotient by one of its prime factors; and so on till a quotient is found that is prime; the several divisors and the last quotient are the required factors.

Exercises for Written Work.

Let it be required to factor 130.

ILLUSTRATION.

2)130
5)65
13

$130 = 2 \times 5 \times 13$

EXPLANATION.—Dividing 130 by 2, we have 65 for a quotient; dividing this quotient by 5, we have 13 for a quotient, which is a prime number; hence, the required factors are 2, 5, and 13.

EXAMPLES.

Resolve the following numbers into prime factors:

1. 210.	*6.* 495.	*11.* 570.	*16.* 342.
2. 330.	*7.* 425.	*12.* 504.	*17.* 824.
3. 1,015.	*8.* 990.	*13.* 1,485.	*18.* 632.
4. 156.	*9.* 975.	*14.* 2,625.	*19.* 548.
5. 310.	*10.* 765.	*15.* 918.	*20.* 350.

EQUAL FACTORS.

The prime factors of 4 are 2 and 2; $2 \times 2 = 4$.

The prime factors of 8 are 2, 2 and 2; $2 \times 2 \times 2 = 8$.

The prime factors of 9 are 3 and 3; $3 \times 3 = 9$.

The prime factors of 16 are 2, 2, 2 and 2.

The number of times the same factor is used in producing a composite number is sometimes indicated by a

figure written at the right and a little above the factor; thus, $3^2 = 9$; $2^3 = 8$; $4^2 = 16$; $2^4 = 16$.

In these illustrations the figure standing above and to the right is called an **exponent**, and the product of the equal factors is called a **power**.

DEFINITIONS.

94. An **Exponent** is a number written at the right and a little above the number, to indicate how many times the number is used as a factor.

95. A **Power** is the product of any number of equal factors. Hence 4 is the second power of 2: and the expression $2^2 = 4$ is read *2 second power*, or *2 square, equals 4*. $2^3 = 8$ is read *2 third power*, or *2 cube*, equals 8.

Read the following

EXAMPLES.

(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)	(*6.*)
$2^2 = 4$,	$2^3 = 8$,	$2^4 = 16$,	$3^2 = 9$,	$3^3 = 27$,	$3^4 = 81$.

What are properties of numbers? What is an exact divisor? What is a prime number? An even number? An odd number? What is factoring? Give the rule for resolving or separating a number into prime factors. What is an exponent? What is a power of a number?

CANCELLATION.

We have learned that multiplying or dividing both divisor and dividend by the same number does not change the value of the quotient.

We can frequently take advantage of this principle to shorten our work.

Divide $29 \times 6 \times 8 \times 3 \times 10$ by $3 \times 6 \times 8$.

ILLUSTRATION.

$$\overset{1}{\cancel{3}} \times \overset{1}{\cancel{6}} \times \overset{1}{\cancel{8}}\ )\ 29 \times \overset{1}{\cancel{6}} \times \overset{1}{\cancel{8}} \times \overset{1}{\cancel{3}} \times 10 = 290.$$

EXPLANATION.—We first divide both divisor and dividend by 3, indicating the division by crossing 3 in each, and writing 1, the quotient of 3 divided by 3, over the factor crossed. In the same manner we divide by 6, and by 8. Our divisor becomes $1 \times 1 \times 1 = 1$, and our dividend becomes $29 \times 1 \times 1 \times 1 \times 10 = 29 \times 10 = 290$.

SECOND ILLUSTRATION.

$$\cancel{645} \div \cancel{105}$$
$$\cancel{129} \div \cancel{21}$$
$$43 \div 7 = 6\tfrac{1}{7}.$$

EXPLANATION.—We divide both divisor and dividend by the common factor 5, and write the quotient beneath. This gives us 21 for a divisor and 129 for a dividend. Again we divide both by the common factor 3, and obtain 7 for a divisor and 43 for a dividend. 43 divided by 7 gives $6\frac{1}{7}$ for the quotient.

96. Cancellation in division is the process of shortening the operation by dividing both divisor and dividend by their common factor, or factors.

RULE FOR CANCELLATION.

Divide both divisor and dividend by all the common factors, crossing the numbers thus divided, writing the quotients over or under the numbers crossed; the product of these quotients and the remaining factors of the dividend, divided by the product of the quotients and remaining factors of the divisor, will give the required quotient.

EXAMPLES.

1. Divide $36 \times 7 \times 14$ by 2×3.

2. Divide $42 \times 8 \times 5 \times 12$ by $7 \times 5 \times 2$.

3. Divide $48 \times 14 \times 3$ by 8×7.

4. Divide $56 \times 18 \times 7 \times 3 \times 5$ by $7 \times 5 \times 2 \times 3$.

5. 390÷78.	*8.* 2625÷1485.	*11.* 23485÷1830.
6. 11850÷2370.	*9.* 5214÷4029.	*12.* 3468÷124.
7. 2910÷2490.	*10.* 2190÷657.	*13.* 2816÷12.

What is cancellation? What is the rule for cancellation? What is the object of cancellation?

GREATEST COMMON DIVISOR.

97. A **common divisor** of two or more numbers is the number that *will exactly divide them separately.*

98. The **greatest common divisor** of two or more numbers is the *greatest number* that will *exactly* divide them separately. Thus, 12 is the greatest common divisor of 24, 36, and 48.

There are two methods of finding the greatest common divisor: ***1.*** *By factors;* and ***2.*** *By continued division.*

METHOD BY FACTORS.

99. When the numbers can be resolved into factors, we may find their greatest common divisor by the following

RULE.

Resolve the numbers into prime factors, and find the product of those that are common to them all.

Let it be required to find the greatest common divisor of 240 and 330.

ILLUSTRATION. 240 resolved into its prime factors $= 2\times2\times2\times2\times3\times5$.

330 resolved into its prime factors $= 2\times3\times5\times11$.

2, 3 and 5 are common prime factors, hence the greatest common divisor $= 2\times3\times5 = 30$.

Find the greatest common divisor of

1. 6, 12, 30.	*4.* 15, 25, 30, 45.
2. 28, 42, 70.	*5.* 2010, 165, 525.
3. 84, 126, 210.	*6.* 3195, 1206.

METHOD BY CONTINUED DIVISION.

100. The greatest common divisor of two numbers can be found, without factoring, by the following

RULE.

I. Divide the greater number by the less; then take the divisor for a new dividend, and the remainder for a new divisor, and proceed as before.

II. Continue this operation till a remainder is found that will exactly divide the preceding divisor; this will be the required greatest common divisor.

ILLUSTRATION.

```
112)144(1
    112
     32)112(3
         96
         16)32(2
```

EXPLANATION.—We divide 144 by 112, and find a remainder 32; we next divide 112 by 32, and find a remainder 16, which exactly divides the preceding divisor; hence, 16 is the greatest common divisor of 112 and 144.

EXAMPLES.

Find the greatest common divisor of the following groups of numbers:

1. 216 and 316.	*4.* 376 and 1645.
2. 39 and 192.	*5.* 1134 and 2079.
3. 1155 and 352.	*6.* 3471 and 1869.

What is a common divisor of two or more numbers? The *greatest common divisor?* How many methods of finding the greatest common divisor? What are they? Give the rule for the method by factors. For the method by continued division.

LEAST COMMON MULTIPLE.

101. A **Multiple** of a number is a number that is exactly divisible by it. Thus, 18 is a multiple of 6.

A **Common Multiple** of two or more numbers is a number that is exactly divisible by each. Thus, 18 is a common multiple of 2, 3, and 6.

The **Least Common Multiple** of two or more numbers is the least number that is exactly divisible by each. Thus, 12 is the least common multiple of 2, 3, and 6.

OPERATION OF FINDING THE LEAST COMMON MULTIPLE.

102. The least common multiple of two or more numbers may be found by the following

RULE.

I. Write the numbers in a line and then divide by any prime factor that is contained in two or more, writing the quotients and also the undivided numbers in the line below.

II. Then operate on this line in the same manner, and so continue till a line is found in which no two numbers have a common factor.

III. Find the continued product of the numbers in the last line and of the divisors used, and it will be the required least common multiple.

ILLUSTRATION.

$$\begin{array}{r|l} 3 & 3,\ 5,\ 6,\ 8 \\ \hline 2 & 1,\ 5,\ 2,\ 8 \\ \hline & 1,\ 5,\ 1,\ 4 \end{array}$$

$$3 \times 2 \times 5 \times 4 = 120$$

EXPLANATION. — Having written the numbers 3, 5, 6 and 8 in a line, we divide 3 and 6 by 3, placing the quotients underneath, and bringing down the undivided numbers; we then divide 2 and 8 by 2, bringing down as before; we then find the continued product of the numbers in the last line and of the divisors used; this gives 120, which is the required least common multiple.

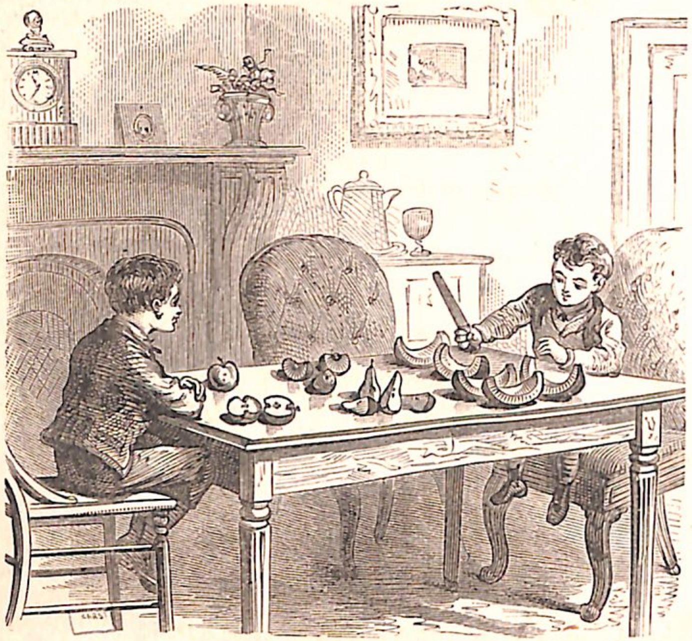
EXAMPLES.

Find the least common multiple of the following groups of numbers:

1. 5, 10, 15, and 20.
2. 10, 15, 24, and 30.
3. 8, 12, 18, and 24.
4. 6, 9, 12, and 15.
5. 15, 36, and 60.
6. 12, 14, 20, and 24.
7. 8, 9, 16, 24, and 27.
8. 7, 8, 15, 21, and 24.

REVIEW QUESTIONS.

What is a factor? What is a composite number? Illustrate. What is a prime number? Illustrate. What is factoring? What is the rule for finding the prime factors of a number? What is cancellation? What is it used for? How may division be simplified by cancellation? What is the greatest common divisor of two numbers? How many methods of finding it? Give the method by factors. By continued division. What is a multiple of a number? What is a common multiple of two or more numbers? The least common multiple of two or more numbers? Give the rule for finding the least common multiple. If the sum of two numbers and one of them is given, how will you find the other? If the difference between two numbers, and the less number be given, how will you find the greater? If the difference between two numbers and the greater be given, how will you find the less?



FORMATION OF FRACTIONS.

103. *1.* How many undivided apples are there in the picture?

What number will represent it?

Write the number by means of a figure.

This is an integer. An integer is a whole number.

2. One of the apples in the picture is divided into two equal parts. What part of the apple is one of the equal parts?

Write in figures the fractional number which will represent one of the halves. Write the fractional number that will represent two halves.

How many halves make one?

3. The peach is divided into three equal parts; what part of the peach is one of the equal parts?

Write in figures *one-third;* write *two-thirds.*

How many thirds make one?

4. The pear is divided into four equal pieces. What part of the pear is one of the pieces? What fraction will express two pieces? What, three pieces? What, four pieces? Write all these *fourths.*

5. The melon is divided into five equal pieces. What part of the melon is each piece?

Write in figures *one-fifth, two-fifths, three-fifths, four-fifths, five-fifths.*

Five-fifths are equal to what integral number?

A **Fraction** is one or more equal parts of a unit.

104. **One** of the equal parts into which an *integral unit* is divided is called a **Fractional Unit.**

In writing fractions we use one of the methods of indicating division, but we call the divisor and dividend by different names. The dividend, or number above the line, we call the *numerator;* the divisor, or number below the line, we call the *denominator.*

105. The **Denominator** shows into how many equal parts the integral unit is divided.

106. The **Numerator** shows how many of these equal parts are expressed by the fraction.

107. The **Value of a Fraction** is the quotient obtained by dividing the numerator by the denominator.

108. The **Terms of a Fraction** are the numerator and the denominator.

What is an integer? A fraction? A fractional unit? An integral unit? What are the terms of a fraction? What is the value of a fraction? What does the denominator show? The numerator? Which corresponds to the dividend? Which to the divisor?

Read the following fractions, and tell which is the numerator and which the denominator, and what each numerator and each denominator shows.

$\frac{7}{8}$, $\frac{13}{20}$, $\frac{12}{13}$, $\frac{5}{99}$, $\frac{9}{17}$, $\frac{7}{50}$, $\frac{7}{63}$, $\frac{3}{100}$, $\frac{7}{25}$, $\frac{8}{21}$, $\frac{4}{70}$.

Examples in Writing Fractions.

1. Write 5 of the 6 equal parts of 1.

2. Write 12 of the 17 equal parts of 1.

3. If the fractional unit is one-twentieth, express 6 fractional units; express, also, 12 and 18 fractional units.

4. If the fractional unit is one-36th, express 32 fractional units; also, 6, 8, 12, 15, 21.

5. If the fractional unit is one-fortieth, express 9 fractional units; also, 16, 25, 69, 75, 36, 40, 18.

6. Write *forty-nine / hundredths.*

7. Write *three hundred and sixty-one / forty-sevenths.*

8. Write *seven thousand six hundred and fifteen / nine hundred and fifteenths.*

9. Write *six thousand four hundred / elevenths.*

10. Write *six thousand two hundred and forty-two / three hundred and fifty-thirds.*

Note.—In the preceding examples, and in all similar examples in this book, the sign / separates the nmmerator from the denominator.

KINDS OF FRACTIONS.

109. A **Proper Fraction** is one in which the numerator is less than the denominator; as, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{8}$.

110. An **Improper Fraction** is one in which the numerator is equal to or greater than the denominator; as, $\frac{4}{3}$, $\frac{7}{4}$, $\frac{8}{2}$, $\frac{9}{9}$.

If the numerator is equal to the denominator, the value of the fraction is equal to 1; thus, $\frac{4}{4} = 1$.

A proper fraction is less than 1; an improper fraction is equal to or greater than 1.

111. A **Mixed Number** is a number composed of a whole number, and a fraction; as, $2\frac{1}{2}$, $5\frac{3}{8}$.

112. A **Simple Fraction** is one in which both terms are whole numbers; as, $\frac{3}{7}$, $\frac{14}{5}$.

113. A **Complex Fraction** is one in which one term, at least, is either a fraction, or a mixed number; as, thus, $\frac{2\frac{1}{2}}{7}$, $\frac{3\frac{1}{4}}{4\frac{1}{2}}$, $\frac{\frac{2}{3}}{5}$ are complex fractions.

114. A **Compound Fraction** is a fraction of a fraction, or several fractions connected by the word of; as, $\frac{1}{2}$ of $\frac{1}{3}$, $\frac{2}{3}$ of $2\frac{1}{2}$, $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{4}{5}$.

115. The **Reciprocal of any number** is 1 divided by that number. The reciprocal of 4 is $1 \div 4$, or $\frac{1}{4}$.

116. The **Reciprocal of any fraction** is 1 divided by that fraction. It is equivalent to the fraction inverted. The reciprocal of $\frac{2}{3}$ is $1 \div \frac{2}{3}$, or $\frac{3}{2}$.

117. **A Fraction is inverted** by causing its terms to change places; thus, $\frac{3}{4}$ inverted is $\frac{4}{3}$.

118. The **Analysis of a fraction** consists in naming its *integral unit*, the *kind* of fraction, its *terms*, its *fractional unit*, the *number of fractional units* and its *value*.

ILLUSTRATION.—In the fraction $\frac{3}{5}$, 1 is its integral unit; it is a simple, proper fraction, 5 is its denominator, and 3 is its numerator, $\frac{1}{5}$ is its fractional unit, 3 is the number of fractional units, and $\frac{3}{5}$ of 1 is its value.

Write the reciprocal of 2; of $\frac{2}{3}$; of $\frac{5}{8}$; of $\frac{3}{4}$; of $\frac{9}{11}$.
Analyze the fractions $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{6}{11}$, $\frac{14}{15}$, and $\frac{17}{15}$.
Invert the fractions $\frac{5}{8}$ and $\frac{3}{7}$, and read them.

PRINCIPLES.

119. From the nature of a fraction, we have the following principles:

1. Multiplying the numerator of a fraction by any number is equivalent to multiplying the fraction by that number.

2. Dividing the numerator of a fraction by any number is equivalent to dividing the fraction by that number.

3. Multiplying the denominator of a fraction by any number is equivalent to dividing the fraction by that number.

4. Dividing the denominator of a fraction by any number is equivalent to multiplying the fraction by that number.

5. Multiplying both terms of a fraction by the same number does not change the value of the fraction.

6. Dividing both terms of a fraction by the same number does not change the value of the fraction.

What is a proper fraction? What is an improper fraction? What is the value of a fraction when the terms are equal? What is a mixed number? What is a simple fraction? What is a complex fraction? What is a compound fraction? What is the reciprocal of a number? What is the reciprocal of a fraction? What is the value of a fraction? What is the analysis of a fraction? State the principles in the order in which they are placed on page 108.

REDUCTION.

120. **Reduction** is the operation of changing the form of a number without altering its value.

The following cases of reduction of fractions depend on the principles given in Art. 119.

121. ***To reduce a fraction to higher terms.***

1. One-half is equal to how many sixths?

SOLUTION.—Since 1 is equal to $\frac{6}{6}$, $\frac{1}{2}$ is equal to $\frac{1}{2}$ of $\frac{6}{6}$, or $\frac{3}{6}$.

2. One-half of an apple is equal to how many fourths?

SOLUTION.—Since 1 is equal to $\frac{4}{4}$, $\frac{1}{2}$ is equal to $\frac{1}{2}$ of $\frac{4}{4}$, or $\frac{2}{4}$.

3. One-fourth of a dollar is how many eighths?

SOLUTION.—Since 1 is equal to $\frac{8}{8}$, $\frac{1}{4}$ is equal to $\frac{1}{4}$ of $\frac{8}{8}$, or $\frac{2}{8}$.

4. Express $\frac{3}{4}$ in terms twice as large.

SOLUTION.—Multiply both terms by 2; thus, $\frac{3\times2}{4\times2}=\frac{6}{8}$.

RULE.

A fraction is reduced to higher terms by multiplying both the numerator and denominator by the same number.

122. ***To reduce a fraction to lower terms.***

We learned in Art. 119, Principle 6, that dividing both

terms of a fraction by the same number does not change the value of the fraction.

1. Reduce $\frac{6}{8}$ to a fraction whose terms are $\frac{1}{2}$ as great.

SOLUTION. $\frac{6\div 2}{8\div 2} = \frac{3}{4}$.

2. How many thirds of an apple are equal to $\frac{8}{12}$ of an apple?

SOLUTION.—Since $\frac{1}{3}$ is equal to $\frac{4}{12}$, there will be as many thirds as there are $\frac{4}{12}$ in $\frac{8}{12}$, or two-thirds.

RULE

For reducing a fraction to lower terms.

Divide both numerator and denominator by the same number.

EXAMPLES.

1. Reduce $\frac{15}{20}$ to an equivalent fraction whose terms shall be $\frac{1}{5}$ as great.

2. How many fourths of a peach are $\frac{6}{12}$ of a peach?

3. Reduce $\frac{25}{35}$ to *sevenths.* Reduce $\frac{15}{25}$ to *fifths.*

4. How many *fifths* are equal to $\frac{36}{45}$?

5. How many *eighths* are equal to $\frac{39}{24}$?

123. *To reduce a fraction to its lowest terms.*

RULE.

I. Resolve the terms into prime factors, and cancel all that are common to both; multiply the remaining factors of the numerator together for a new numerator, and the remaining factors of the denominators for a new denominator; or,

II. Divide both terms of the fraction by their greatest common divisor.

EXAMPLES.

1. Reduce $\frac{12}{18}$ to its lowest terms.

SOLUTION. $\frac{12}{18} = \frac{\cancel{2}\times 2\times \cancel{3}}{\cancel{2}\times \cancel{3}\times 3} = \frac{2}{3}$. *Ans.*

2. Reduce $\frac{24}{60}$ to its lowest terms.

3. Reduce $\frac{35}{135}$ to its lowest terms.

4. Reduce the following fractions to their lowest terms:

5. $\frac{132}{180}$.	*8.* $\frac{39}{243}$.	*11.* $\frac{1008}{875}$.	*14.* $\frac{942}{21}$.
6. $\frac{138}{150}$.	*9.* $\frac{96}{144}$.	*12.* $\frac{616}{504}$.	*15.* $\frac{840}{40}$.
7. $\frac{805}{875}$.	*10.* $\frac{330}{78}$.	*13.* $\frac{330}{858}$.	*16.* $\frac{384}{32}$.

If the terms cannot be factored by inspection, work by the second rule.

17. $\frac{1872}{2016}$.	*22.* $\frac{1089}{1573}$.	*27.* $\frac{3168}{5148}$.
18. $\frac{3069}{3168}$.	*23.* $\frac{1309}{1785}$.	*28.* $\frac{1050}{1386}$.
19. $\frac{363}{605}$.	*24.* $\frac{350}{1015}$.	*29.* $\frac{3508}{3876}$.
20. $\frac{2587}{3383}$.	*25.* $\frac{805}{875}$.	*30.* $\frac{4950}{7700}$.
21. $\frac{5661}{6327}$.	*26.* $\frac{504}{1845}$.	*31.* $\frac{1628}{2244}$.

What is reduction? How is a fraction reduced to higher terms? How is a fraction reduced to lower terms? Give the rule for reducing fractions to their *lowest* terms.

124. *To reduce an improper fraction to a whole or mixed number.*

How many integral units are there in $\frac{12}{2}$?

ILLUSTRATION.—Since 1 is equal to $\frac{2}{2}$, $\frac{12}{2}$ are equal to as many ones as there are $\frac{2}{2}$ in $\frac{12}{2}$, equal to 6.

RULE.

Divide the numerator by the denominator.

EXAMPLES.

1. Reduce $\frac{7}{3}$ to a mixed number. *Ans.* $2\frac{1}{3}$.

2. Reduce $\frac{38}{7}$ to a mixed number. *Ans.* $5\frac{3}{7}$.

Reduce the following to mixed numbers:

3.	$\frac{41}{9}$.	*7.*	$\frac{223}{15}$.	*11.*	$\frac{222}{28}$.
4.	$\frac{311}{12}$.	*8.*	$\frac{442}{7}$.	*12.*	$\frac{302}{11}$.
5.	$\frac{480}{11}$.	*9.*	$\frac{752}{13}$.	*13.*	$\frac{849}{8}$.
6.	$\frac{326}{13}$.	*10.*	$\frac{75}{22}$.	*14.*	$\frac{72413}{720}$.

125. ***To reduce an integer to a simple fraction having a given denominator.***

In any number there are twice as many halves as whole ones, three times as many thirds, four times as many fourths, etc.

1. How many halves in 2 apples?

ILLUSTRATION.—Since in 1 apple there are $\frac{2}{2}$, in 2 apples there are twice $\frac{2}{2} = \frac{4}{2}$.

2. In 5 bushels of wheat, how many thirds?

ILLUSTRATION.—Since in 1 bushel there are $\frac{3}{3}$, in 5 bushels there are five times $\frac{3}{3} = \frac{15}{3}$.

3. Reduce 8 to a fraction whose denominator is 5.

ILLUSTRATION.—In 1 there are $\frac{5}{5}$, in 8 there are 8 times $\frac{5}{5} = \frac{40}{5}$.

From these illustrations we deduce the following

RULE.

Multiply the integer by the given denominator and write the product over that denominator.

EXAMPLES.

4. Reduce 12 to *thirds.* Reduce 9 to *halves.*

5. Reduce 9 to *eighths.* Reduce 7 to *thirds.*

6. Reduce 12 to *sixteenths;* 13 to *fourths.*

7. Reduce 5 to *fifteenths;* 15 to *fifths.*

8. Reduce 14 to *eighteenths;* 17 to *ninths.*

9. Reduce 75 to *fifths;* 84 to *thirds.*

10. Reduce 115 to *fourths;* 112 to *sixths.*

11. Reduce 86 to *ninths;* 73 to *eighths.*

126. ***To reduce a mixed number to an improper fraction.***

Let it be required to reduce $4\frac{7}{8}$ to *eighths.*

ILLUSTRATION.

$4 = \frac{32}{8}$, hence

$$4\frac{7}{8} = \frac{32}{8} + \frac{7}{8} = \frac{39}{8}$$

EXPLANATION.—$4=\frac{32}{8}$, hence, $4\frac{7}{8}=\frac{32}{8}$ and $\frac{7}{8}$; but 32 *eighths* and 7 *eighths* make 29 *eighths,* that is, $4\frac{7}{8}=\frac{39}{8}$. Here we have multiplied 4 by 8 and to the product we have added 7; we have then written the sum over 8. Hence the

RULE.

Multiply the integral part by the denominator of the fractional part, add the numerator to the product, and write the sum over the denominator.

EXAMPLES.

1. Reduce $7\frac{1}{8}$ to an improper fraction. *Ans.* $\frac{57}{8}$.

2. Reduce $8\frac{1}{4}$ to an improper fraction. *Ans.* $\frac{33}{4}$.

3. Reduce $11\frac{3}{7}$ to an improper fraction. *Ans.* $\frac{80}{7}$.

Reduce the following mixed numbers to improper fractions:

4.	$102\frac{5}{9}$.	*7.*	$49\frac{1}{8}$.	*10.*	$97\frac{5}{16}$.
5.	$236\frac{4}{10}$.	*8.*	$63\frac{4}{11}$.	*11.*	$84\frac{3}{20}$.
6.	$215\frac{5}{14}$.	*9.*	$88\frac{3}{14}$.	*12.*	$114\frac{21}{44}$.

13. How many *twelfths* in $18\frac{2}{12}$? In $21\frac{5}{12}$? In $35\frac{7}{12}$?

14. How many *fifteenths* in $17\frac{3}{15}$? In $27\frac{4}{15}$? In $36\frac{5}{15}$?

127. *To reduce fractions to equivalent fractions having a common denominator.*

RULE.

Multiply both terms of each fraction by the product of the denominators of all the other fractions.

EXAMPLES.

1. Reduce $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ to equivalent fractions having a common denominator. Reduce $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{7}$.

2. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ to equivalent fractions having a common denominator. Reduce $\frac{2}{5}$, $\frac{3}{7}$, and $\frac{4}{9}$.

3. Reduce $\frac{3}{8}$, $\frac{7}{11}$, $\frac{8}{13}$, and $\frac{5}{7}$ to equivalent fractions having a common denominator. Reduce $\frac{5}{8}$, $\frac{6}{11}$, $\frac{7}{10}$, and $\frac{4}{15}$.

4. Reduce $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{3}{4}$ each, to twelfths.

5. Reduce $\frac{7}{9}$, $\frac{3}{5}$, $\frac{6}{7}$, $\frac{9}{11}$, $\frac{10}{13}$ to equivalent fractions having a common denominator.

The common denominator is any *multiple of all the denominators.* The *least common denominator* is the least multiple of the denominators.

1. Reduce $\frac{2}{3}$, $\frac{5}{7}$, and $\frac{3}{14}$ to equivalent fractions having the least common denominator.

ILLUSTRATION.

$$7\underline{)3,\quad 7,\quad 14}.$$
$$\quad 3,\quad 1,\quad 2$$
$$7\times3\times2=42.$$

$$\tfrac{2}{3}=\tfrac{28}{42}\qquad \tfrac{5}{7}=\tfrac{30}{42}\qquad \tfrac{3}{14}=\tfrac{9}{42}$$

EXPLANATION.—Find the common multiple of the denominators, which is 42. One-third of 42 is 14. Now, if 3, the denominator of the first fraction, is multiplied by 14, we obtain 42; but if the denominator is multiplied by 14, the numerator must be multiplied by the same number to preserve the value; hence we have $\frac{2}{3}=\frac{28}{42}$. In the same manner $\frac{5}{7}$ and $\frac{3}{14}$ are reduced to *forty-seconds.*

128. *Fractions may be reduced to equivalent fractions having the least common denominator by the following*

RULE.

Find the least common multiple of all the denominators for a common denominator; then multiply both terms of each fraction by the quotient of the least common multiple by the denominator of that fraction.

EXAMPLES.

Reduce the following groups to their least common denominator:

2. $\frac{3}{5}$, $\frac{4}{9}$, and $\frac{7}{18}$. *Ans.* $\frac{54}{90}$, $\frac{40}{90}$, $\frac{35}{90}$.

3. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{10}$. *Ans.* $\frac{20}{30}$, $\frac{24}{30}$, $\frac{25}{30}$, and $\frac{21}{30}$.

4. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{10}$, and $\frac{2}{9}$. *Ans.* $\frac{60}{90}$, $\frac{75}{90}$, $\frac{63}{90}$, and $\frac{20}{90}$.

5. $\frac{2}{5}$, $\frac{3}{8}$, and $\frac{5}{16}$.

6. $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$.

7. $\frac{7}{25}$, $\frac{11}{30}$, and $\frac{8}{15}$.

8. $\frac{2}{15}$, $\frac{5}{16}$, and $\frac{11}{20}$.

9. $\frac{7}{9}$, $\frac{11}{15}$, and $\frac{11}{12}$.

10. $\frac{3}{5}$, $\frac{2}{9}$, $\frac{2}{3}$, and $\frac{7}{15}$.

11. $\frac{2}{27}$, $\frac{3}{54}$, $\frac{7}{9}$, and $\frac{5}{18}$.

12. $\frac{5}{32}$, $\frac{3}{28}$, and $\frac{11}{20}$.

If fractions have a common denominator, they have the same fractional unit.

TEST QUESTIONS.

What is a fraction? What is a proper fraction? What is a simple fraction? What is an improper fraction? How is an improper fraction reduced to a whole or mixed number? What is a mixed number? How is a mixed number reduced to an improper fraction? Give the rule for reducing an integer to a simple fraction having a given denominator. How are fractions having different denominators reduced to equivalent fractions having a common denominator? Give the rule for reducing fractions to equivalent fractions having the least common denominator. What is a complex fraction?